Design Qualifier Examination (New Format)

May 21, 2013

Answer any 4 out of 5 questions.

Time: 180 minutes

All questions carry equal marks

Pledge: *On my honor, I have neither given or taken assistance on this examination*

Signature________________________________

1. Optimality Conditions

   a) Consider the following minimization problem:

   Minimize

   $$F(\bar{x}) = (x_1 - 10)^2 + (x_2 - 8)^2$$

   s.t.

   $$x_1 + x_2 \leq 12$$
   $$x_1 \leq 8$$

   Develop the KKT optimality conditions for this problem.

   b) *Using your conditions* from part a), find the optimal solution for the problem.

   c) Verify mathematically that this solution is a global minimum.

   d) Validate the solution graphically.

   e) An optimization problem has one equality constraint, h, and one inequality constraint, g. It is believed that there is a solution with the following information:

   $$h = 0,\ g = 0,\ \nabla f = (2,3,2),\ \nabla h = (1,-1,1),\ \nabla g = (-1,-2,-1)$$

   Confirm or deny this belief using the KKT conditions.

2. Steepest Descent Search

   a) Consider the following minimization problem:

   Minimize

   $$F(\bar{x}) = x_1^2 + x_2^2 - 2x_1x_2$$

   Solve this problem using the steepest-descent method starting from the point $x_0 = (1,0)$. Clearly state the assumptions you make in finding your solution.

   b) Does your solution satisfy the necessary and sufficient conditions for optimality? Show your work.

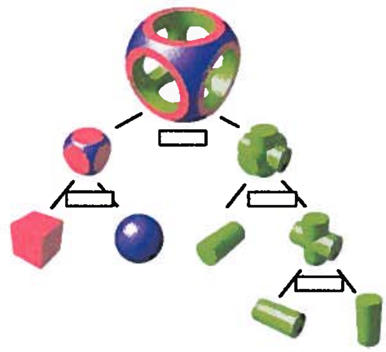
   c) Now assume that two inequality constraints are added to this problem:

   $$g_1 : x_1^2 - x_2 \geq 1$$
   $$g_2 : x_1 + 3x_2 \leq 9$$

   Describe how you would now solve this optimization problem assuming that you still wanted to use the steepest-descent method. You do not need to solve the problem. Instead, clearly describe how you would formulate the problem and apply the steepest-descent method.

3. a) The following data points $P_0(1,5)$, $P_2(2,6)$, $P_3(3,7)$, and $P_4(6,1)$ were collected through a coordinate measuring machine. Propose a technique to find control points for a Bazier curve which passes exactly or very close to these experimental data points. Demonstrate that your technique works by generating four sample data points. Also address the following: How many control points would be needed? What would be the degree of this curve?

   b) Briefly describe the fundamental principles of Constructive Solid Geometry (CSG) technique used to obtain 3D solid model. What operations would you perform to obtain the following solid object (fill in the rectangular boxes)?



4. Find the minimum value/point of the following function using
   a. Lagrange's method
   b. Penalty function method

$$\begin{aligned} &\textit{minimize } f(x) = 0.03(x_1 - 2x_1^2) + x_2^2 - 1.072 \\ &\textit{Subject to} \quad x_2^2 - (0.0234x_1 + 0.075x_1^2 - 1.788)^2 = 0 \\ &\qquad\qquad\quad x_2 + 0.09x_1 + 0.089x_1^2 - 1.162 \geq 0 \end{aligned}$$

Use the starting point $\mathbf{x} = [1,1]^T$

5. Do the following 2D transformations manually and clearly show all the steps in calculations (Matrix operations). Draw a neat figure for all the following 2D transformations. Consider a 2D line with end points (0, 0) and (3, 4)
   a) Translate it by +5 units in X and Y directions.
   b) Scale the line by a factor of +3.
   c) Rotate the line by 35° in anticlockwise direction.
   d) Find the reflection of the given line around X axis.