SYSTEMS AND DESIGN – QUALIFYING EXAM – MAY 24, 2011

Answer any 4 out of 6 questions from Systems and Design/Dynamic Systems Exam
No Books - Calculators allowed

DESIGN

Problem 1

On a two-dimensional map, two sensors have been placed at (0,0) and (3,2). You are in charge of determining the best location to place a third sensor that minimizes the sum of the squares of the distances from the third sensor to the first and second sensors. The third sensor must reside on the x-axis.

a) Formulate this problem as a standard optimization problem.
b) Using the origin as the starting point, carry out one iteration of Newton's Method to solve the problem. Show your work clearly.
c) State the optimality conditions for the problem you formulated in part (a) and then show if your solution from part (b) satisfies these conditions. Explain your findings.
d) What other methods would be appropriate to solve your problem from (a)? Explain why these methods are appropriate.

Problem 2

Given the following constrained optimization problem:

Min.

$$F(\vec{x}) = (x_1 - 3)^2 + (x_2 - 4)^2$$

s.t.

$$g_1(\vec{x}) : x_2 \le 3$$
$$g_2(\vec{x}) : x_1 \le 2$$
$$g_3(\vec{x}) : x_2 - x_1 \le 3$$

$$x_i \ge 0$$

a) Sketch contours of the objective function and graph the constraints.
b) Identify the optimal point graphically and show analytically that it satisfies the optimality conditions for this problem.
c) If you were to use a SUMT method to model this problem analytically, formulate your new objective function and explain what solution approach you would use to find the optimum, clearly supporting your choice. You do not have to actually solve the problem analytically.

Problem 3

Bezier curves are used to represent letters and other symbols in postscript fonts in printers. Show a set of control points to define a Bezier curve that gives the representation of letter C or S.

Address the following:

a) How many control points would be needed?
b) What would be the degree of this curve?
c) Select a set of sample control points and show representative points on the font (C or S) to show that your selection of control gives a decent font using Bezier curve.

SYSTEMS

Problem 4

Consider the following matrix:

$$A = \begin{bmatrix} 1 & \alpha & \alpha & \alpha \\ 0 & 1 & 0 & \alpha \\ 0 & 0 & 1 & \alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where α is a real constant.

a) What are the eignevalues of A?

b) Show the diagonal or Jordan block form (note, you do not need to prove your result is correct, just give the final form).

Problem 5

Consider the following matrix:

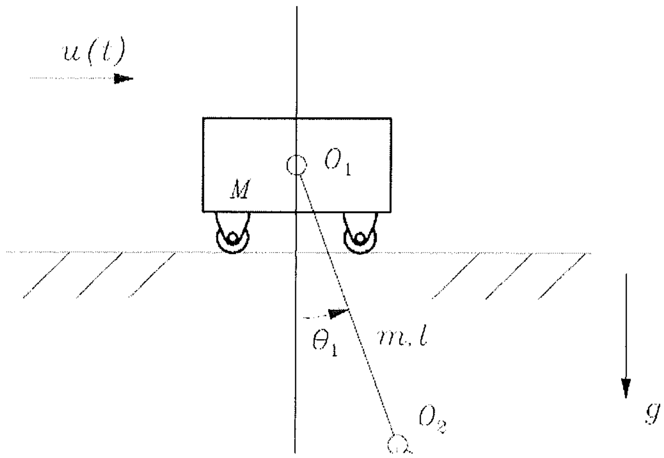
$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

a) Find the characteristic equation and eigenvalues of this matrix.

b) Use the Cayley-Hamilton Theorem to determine e^{At}.

c) What is the inverse of e^{At}?

Problem 6



The system at hand is an inverted pendulum mounted on a horizontally translating base. The inverted pendulum can be modeled as a "slender rod" (uniform mass, c.m. at midpoint) of length l and mass m with the base joint O_1 and is parameterized by the angle $\theta_1(t)$. It is mounted on a base of mass M that can slide under controlled motion, parameterized by the linear variable u(t) along a horizontal track. The two joints are lubricated with a fluid that produces linear viscous damping (i.e. proportional to joint-velocity) and the same joints are acted upon by motors producing driving forces/torques (F and τ).

(i) Determine the dynamics model of the system under the action of gravity and the equilibrium configuration(s).

(ii) Develop a linearized dynamic model about a stable equilibrium configuration and express it in a state-space form.