Department of Mechanical and Aerospace Engineering
Systems and Design
2009 Ph.D. Written Qualifying Exam
Thursday May 14th 2009

No textbooks or notes allowed. You are required to answer 4 out of 6 questions

Assigned # ___________

Problem 1

Consider the following state-space system:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} a \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} b & 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \end{bmatrix} u$$

a) What are the eigenvalues of the system?
b) Is the system stable?
c) What is the matrix exponential of the state matrix?
d) What values of a make the system uncontrollable?
e) What values of b make the system unobservable?

Problem 2

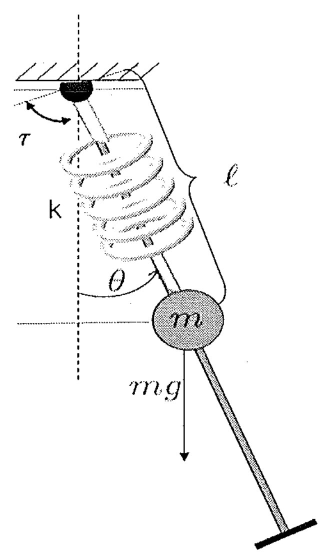
Find a similarity matrix that transforms the following matrix into Jordan form:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

What is the final solution of the Jordan form for this system?

Problem 3

Derive the equations of motion of the mass (m) sliding on a rigid massless shaft which rotates about a pivot by an angle θ and is driven by a torque τ. The mass is attached by a spring of stiffness (k) to the pivot and is subject to gravity.



Problem 4
Given the optimization problem:

$$\begin{aligned} Min:\ & F(\vec{x}) = x_1^2 + x_2^2 \\ s.t.\ & g_1(\vec{x}) = 5x_1 + x_2 \geq 5 \\ & g_2(\vec{x}) = 3x_1 + 2x_2 \geq 6 \end{aligned}$$

and the current design point of $x_1 = 1\frac{5}{13}$ and $x_2 = \frac{12}{13}$

a) Show whether this point satisfies the Kuhn-Tucker conditions.
b) Find the Lagrange multiplier values.
c) Explain the significance of these Lagrange multiplier values. Also, explain what significance the multiplier values have in sensitivity analysis of this problem. Be as specific as possible.
c) How would you classify this candidate design point? Again, be specific as possible using information from the problem formulation in your answer.

Problem 5
Consider (i.e., do not solve) the root-finding problem below in the following questions.

Find the root of $F(\vec{x}) = 5x^2 + 4x - 1$ within $x \in (-2,0)$

a) Explain how you could formulate this problem as an optimization problem. Show your formulation.
b) Using $x_0 = 0$, demonstrate how you would use Steepest Descent to solve this problem. Conduct two iterations.
c) List one zero-order methods that could be used to solve your optimization problem.
d) Explain the significance of 1-D searches in the context of multidimensional numerical optimization processes.
e) In Figure O1 below, identify the method being used. Clearly show and explain where the 1-D searches are being used in the figure (ignore the X^o in the top right).
f) In Figure O2 below, identify the first two possible iterations using Method of Feasible Directions. You do not have to be exact. Again, clearly show and explain where the 1-D searches are being used in the figure.

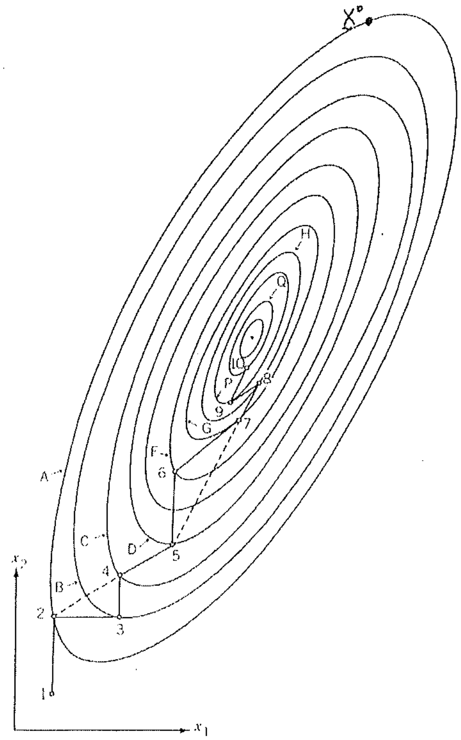


Figure O1

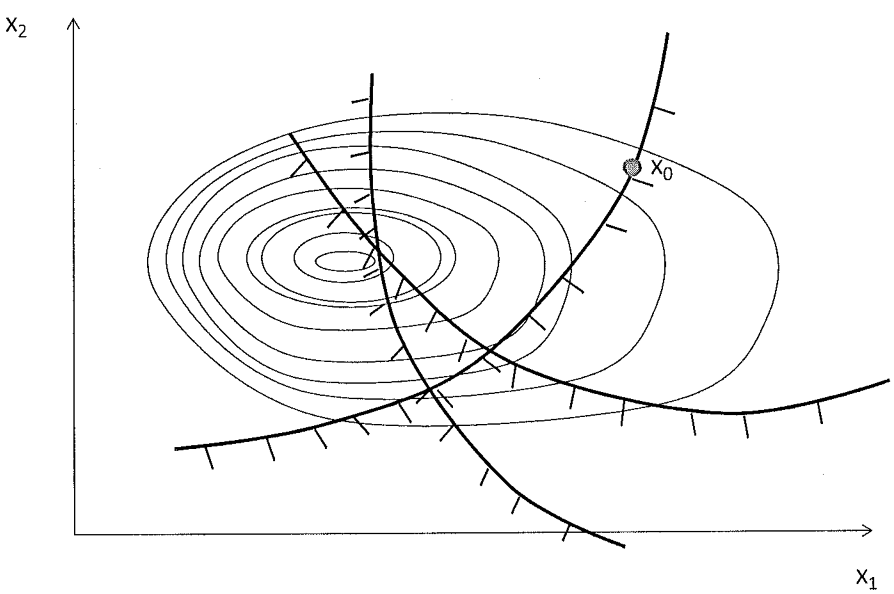


Figure O2

Problem 6

A tapered elastic bar has length L and cross-sectional area $A(x)$, as shown in the figure below. Assume that the response is governed by the following one-dimensional differential equation of equilibrium:

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right)+f=0,$$

where E is the elastic modulus, $u(x)$ is the axial displacement and $f(x)$ is an applied body force per unit length. The bar is restrained at $x=0$, while normal tractions are applied at $x=L$ as illustrated, such that

$$EA\frac{du}{dx}\bigg|_{x=L}=P.$$

Derive the principle of virtual work for this bar and develop a two-noded bar element that could be used to find approximate solutions.