University at Buffalo
Department of Mechanical & Aerospace Engineering

Thermal-Fluids
Ph.D. Qualifying Exam

21 May 2013

Problem #1

A large wind turbine with diameter D extracts a fraction η of the kinetic energy from the airstream (density = ρ= constant) that impinges on it with velocity U.
a) What is the diameter of the wake zone, E, downstream of the windmill?
b) Determine the magnitude and direction of the force on the windmill in terms of ρ, U, D, and η.

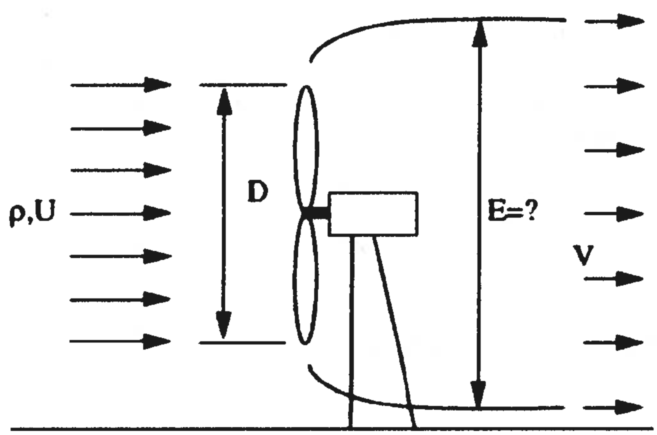

Figure 1

Problem #2

A 2-D vortex in the x-y plane consists of a circular region of radius R in which the vorticity is constant and equal to $\vec{k}\Omega$. Outside this region the vorticity is zero.

a) Determine the velocity distribution in the region r<R and in r>R.
b) The complex potential of a rectilinear vortex, which is a representation of the vortex described above for r>>R is given by $w = -i\Lambda \ln z$. Find the relation between the vortex strength Λ and the parameters R and Ω.

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Problem #3

Viscous incompressible fluid with density ρ and viscosity μ is in an infinitely long annulus (Fig. 2) $a \le r \le b$. The outer cylinder rotates at with angular speed Ω while the inner cylinder translates along its own axis with speed W. The Navier-Stokes and continuity equations in cylinderical coordiantes are:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial(\rho u_\phi)}{\partial \phi} + \frac{\partial(\rho u_z)}{\partial z} = 0.$$

$$r: \ \rho\left(\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_r}{\partial \phi} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\phi^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \phi^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\phi}{\partial \phi}\right] + \rho g_r$$

$$\varphi: \ \rho\left(\frac{\partial u_\varphi}{\partial t} + u_r\frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi} + u_z\frac{\partial u_\varphi}{\partial z} + \frac{u_r u_\varphi}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \varphi} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r u_\varphi)\right) + \frac{1}{r^2}\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \frac{\partial^2 u_\varphi}{\partial z^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \varphi}\right]$$

$$z: \ \rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\phi}{r}\frac{\partial u_z}{\partial \phi} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \phi^2} + \frac{\partial^2 u_z}{\partial z^2}\right] + \rho g_z.$$

a) Simplify the equations under the assumption that the flow is steady, no body forces, and no axial pressure gradient. Describe any additional assumptions that you feel relevant and simplify accordingly.

b) Non-dimensionalize the equations choosing the approparaite scales for all variables and pose apporpariate boundary conditions.

c) Solve the resulting equations to determine the velocity field.

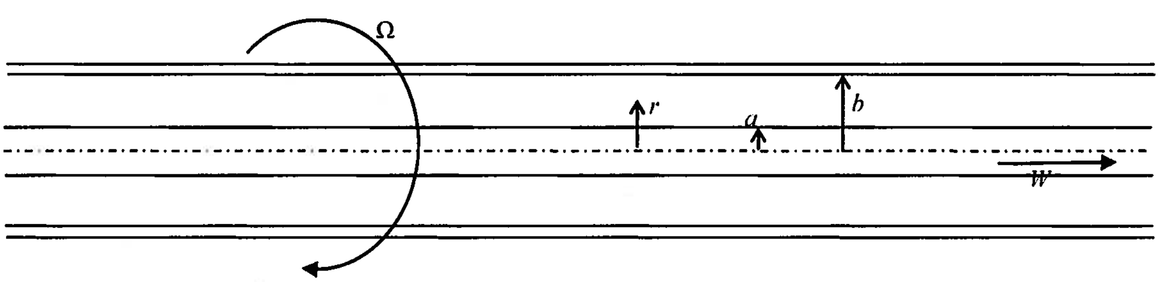

Figure 2

Heat Transfer Qualifying Exam **May 21, 2013**
2013

Instructions:

1. You will have 90 minutes.
2. Carefully read each problem before beginning the problem.
3. Show all work.
4. No books or notes.
5. Define all quantities.
6. All answers must be circled.

Problem 1 (??%) Starting with the equation $Tds = du + pdv$ obtain the simplest expression for the entropy change of an ideal gas during a process $1 \rightarrow 2$ assuming constant specific heats.

Problem 2 (??%) Consider the following steady-state, steady-flow reaction:

$$CH_4+2O_2 \rightarrow CO_2+2H_20(l).$$

The reactants and products are each at a total pressure of 0.1 MPa and 25°C. Determine the heat transfer per kilomole of fuel entering the combustion chamber. Make sure that you indicate what each term in the final answer is.

Problem 3 Consider a cycle with the following process:

1 $\rightarrow$ 2 Isentropic compression

2 $\rightarrow$ 3 Constant volume heat addition

3 $\rightarrow$ 4 Isentropic expansion

4 $\rightarrow$ 1 Constant pressure heat rejection

Problem 3(a) (??%) Assuming constant specific heats determine the total work per mass of this cycle in terms of specific heats and temperatures.

Problem 3(b) (??%) What is the expression for the thermal efficiency of this cycle? You may leave the efficiency in terms of temperatures and specific heat constants.

STATE UNIVERSITY OF NEW YORK AT BUFFALO
DEPARTMENT OF MECHANICAL & AEROSPACE ENGINEERING

Fluid Thermal Sciences
PhD Qualifying Examination
May 21, 2013

********** THREE (3) HOURS **********

********** CLOSED BOOK / NOTES / ETC. **********

1) Fundamentals:
   a) Describe (using words) the modes of heat transfer with emphasis on the physics of the modes
   b) Write the basic rate equations for the modes of heat transfer with all symbols used clearly defined
   c) Make a list of all of the assumptions embodied in the rate equations in 1b
   d) Give an example of a 1^{st} Law Control Volume analysis
2) Conduction:
   a) Write the general Heat Conduction Equation with all symbols used clearly defined and all assumptions listed
   b) Solve the Heat Conduction Equation for any problem of your choice using suitably prescribed boundary and/or initial conditions
3) Convection:
   a) Write the governing equations for forced convection heat transfer with all symbols used clearly defined and all assumptions listed
   b) Solve the governing equations for forced convection heat transfer for any problem of your choice using suitably prescribed boundary and/or initial conditions
4) Radiation:
   a) Write the governing equations for radiation heat transfer with all symbols used clearly defined and all assumptions listed
   b) Solve the governing equations for radiation heat transfer for any problem of your choice using suitably prescribed boundary and/or initial conditions
5) Phase change:
   a) Write the governing equations for phase change heat transfer with all symbols used clearly defined and all assumptions listed
   b) Solve the governing equations for phase change heat transfer for any problem of your choice using suitably prescribed boundary and/or initial conditions