May 24, 2011

ASSIGNED NUMBER ________ Name______________________________

MAE Ph.D. Qualifying Exam: Part I – Fluid Mechanics
(90 minutes for Part I, **CLOSED BOOK**)

1. Potential flow.

For the following, assume steady, incompressible, two-dimensional flow.

a. Show that a flow which can be described using the *velocity potential*, ϕ, must be irrotational.

b. Show that ϕ satisfies Laplace's equation.

c. Consider the *stream function*, ψ. Show that ψ also satisfies Laplace's equation for a potential flow.

d. The radial and azimuthal velocity components for a *potential vortex* (2-D) are $u_r = 0$ and $u_\theta = \dfrac{A}{2\pi r}$, respectively. Show that the circulation, Γ, of the vortex is A.

e. How is it possible that the "potential" vortex has a non-zero circulation? Is the flow irrotational everywhere? Why or why not?

f. Find the pressure distribution in the potential vortex, relative to the far-field pressure, p_∞. Is the pressure at/near $r = 0$ physically possible? Why or why not?

2. Double falling film on a wall.

Consider the flow shown in figure 2, where a smooth plane wall is inclined at an angle θ to the vertical. Two immiscible liquid films flow down the wall under the influence of gravity. At the instant shown in the figure, the flow is already fully-developed. Above the film is air at atmospheric pressure. Use the rectangular coordinate system shown.

Assumptions: incompressible flow; the thicknesses of the bottom and top films, h_a and h_b, respectively, are known and constant; there is only one non-zero velocity component, which is a function of y alone; the density of liquid a, ρ_a, is greater than that of liquid b, i.e. $\rho_a > \rho_b$; the viscosities are such that $\mu_a < \mu_b$.

a. Find the pressure at the liquid-liquid interface. What is it when $\theta = 0$?

b. Write down the x-component of the full, incompressible Navier-Stokes equations *without* assumptions.

c. Simplify the equation in part (b) as appropriate.

d. Solve for the velocity in each layer, but *do not* apply boundary conditions (i.e. find general expressions).

e. How many boundary conditions are required to solve for the flow and what are they?

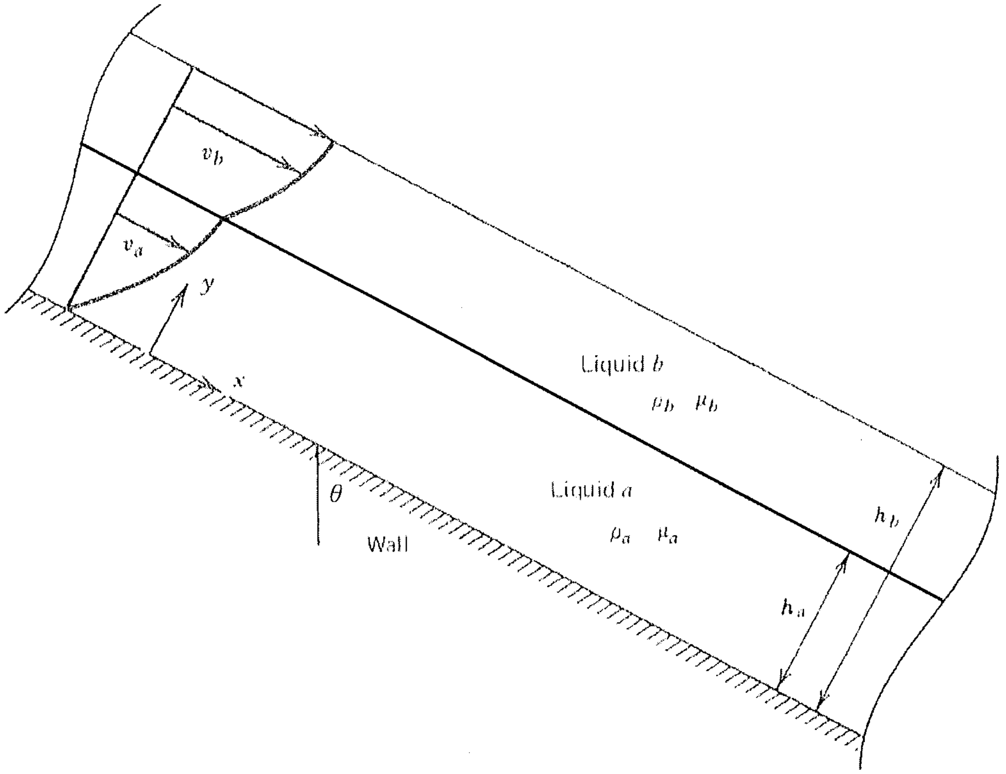

Figure 2. Films of two immiscible fluids falling down and inclined wall.

3. Flow kinematics.
   a. Consider a 2-D stagnation flow, where the x-component of velocity is $u = cx$, and the y-component is $v = -cy$. Sketch the flow (qualitatively). What is the out-of-plane component of vorticity, ω_z?

   b. Consider plane Couette flow in the x-direction (between infinite parallel plates separated by a gap h, where the top plate moves with a velocity U in the x-direction and the bottom plate is stationary). Sketch the flow between the plates. What is ω_z?

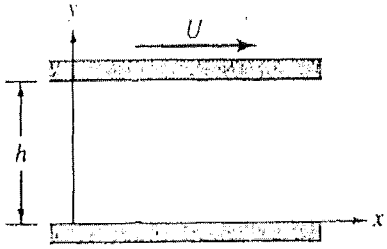

Figure 3b. Geometry for plane Couette flow, walls are infinite.

   c. Based on your answers to parts (a) and (b), is there a direct connection between vorticity and streamline curvature? Please explain.

d. The vorticity vector, $\vec{\omega}$, is divergence free, i.e. $\nabla \cdot \vec{\omega} = 0$. Using this, show that the circulation, Γ_1, around a given cross-section of a *vortex tube* is the same as that at some other cross-section, Γ_2, i.e. $\Gamma_1 = \Gamma_2$.

e. Consider a vortex tube connected to itself end-to-end (like a donut), assume inviscid flow. Does the circulation, Γ, around the tube change with time? Please explain.

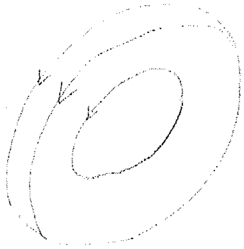

Figure 3e. Vortex tube connected to itself, lines are vortex lines.

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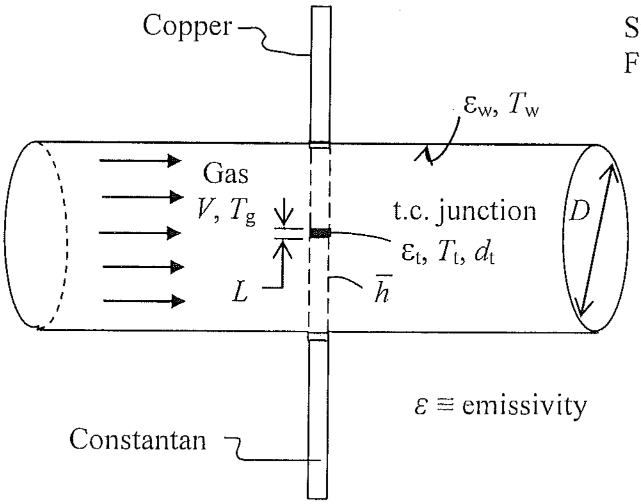
HEAT TRANSFER
(90 min., Closed Book)

Thermocouple Error



Steady-state
Fully-developed hydrodynamically
and thermally

$(T_g - T_t) = ?$

A thermocouple (t.c.) junction of diameter d_t and length L ($L << d_t$) has been designed as shown above. Its purpose is to infer the temperature of the gas flowing through the duct. Calibrations, however, always indicate a difference between the gas temperature, T_g, and the temperature measured by the thermocouple, T_t. The objective of the analysis is to determine the magnitude of this '*thermocouple error*', $(T_g - T_t)$.

Assume that the flow inside the tube is steady, fully-developed hydrodynamically and thermally, and that all material properties are known. In addition, the wall temperature, T_w, thermocouple temperature, T_t, and mean air speed, V, have been measured.

(a) upon what variables would the mean heat transfer coefficient between the gas and the thermocouple, $\bar{h}$, depend?

(b) given that $\bar{h}$ can be estimated, derive the equations and boundary conditions which determine the temperature distributions along the thermocouple wires and the gas temperature T_g ;

(c) neglecting heat conduction down the wires, what is $(T_g - T_t)$?

State any assumptions you make.

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Short-Pulse Heating of Metals

(a) Describe the thermal response within a metal when it is exposed to very short pulses from, say, a picosecond laser.

(b) Write two (coupled) equations which model this behavior.

Heat Conduction across Thin Dielectric Films

(a) When the phonon mean free path in a dielectric material is much smaller than the thickness of the material, will the thermal conductivity be larger or smaller than that of the bulk material. Why?

(b) Write an expression for the heat flux across such a thin film.

Volumetric Heat Source due to 'Joule Heating', $\dot{q}_e$

Let: φ = voltage
r_e = electrical *resistivity* (ohm-m)
$\overline{j}$ = flux of electrical charge (charge/area-time)

For a steady, 3-D flow of charge through a material, derive the following results:

(a) $\nabla \cdot (r_e^{-1} \nabla \varphi) = 0$

(b) $\dot{q}_e = j_e^2 r_e$, net flow of electrical energy *into* $\delta m = \rho \delta V$