Part I - Fluid Mechanics

1) Given the following velocity field for steady, incompressible flow,
\[ \vec{V} = Ax \hat{i} - Ay \hat{j}, \]
a. Determine the stream function that will yield this velocity field.
b. Plot the streamline pattern in the first quadrant of the xy plane.
c. What type of flow does the streamline pattern represent?
d. Does the velocity field satisfy continuity? Why or why not?
e. Is the flow irrotational or rotational and why?
f. Determine the velocity potential function.
g. Show that the lines of constant potential function and streamlines are orthogonal.

2) A fluid is at rest between two infinite, stationary flat plates.

At time \( t = 0 \) the velocity \( V(y,0) = 0 \). At time \( t = 0^+ \) both plates are suddenly accelerated to and then maintained at a velocity \( V(h,t) = V(0,t) = V'_o \).

Find the velocity distribution in the fluid \( V(y,t) \).
A thermocouple (t.c.) junction of diameter $d_t$ and length $L$ has been designed as shown above. Its purpose is to measure the temperature of the air flowing through the duct. Calibrations, however, always indicate a difference between the air and the thermocouple temperatures. The objective of this analysis is to determine the reason for this ‘thermocouple error’ and calculate its value: $(T_1 - T_a)$.

**Known:**
- thermocouple diameter, $d_t = 5 \text{ mm}$
- thermocouple length, $L = 3 \text{ mm}$
- thermocouple emissivity, $\varepsilon_t = 0.6$
- mean air speed, $V = 5 \text{ m/s}$
- nominal air temperature, $T_a = 500^\circ \text{C}$
- duct diameter, $D = 30 \text{ cm}$
- duct wall temperature, $T_w = 100^\circ \text{C}$
- duct wall emissivity, $\varepsilon_w = 0.8$

1. a. identify (and write) the heat transfer mechanisms which must be included in a mathematical model of the thermal behavior of the system:
   b. perform 1st Law analyses on (1) the junction and (2) the t.c. wires. and derive relations which could be used to determine the thermocouple error, $(T_1 - T_a)$;

2. simplify the model (and list all assumptions) to enable a straightforward computation of $(T_1 - T_a)$ and then compute $(T_1 - T_a)$ for the given: $T_1, V, d_t, L, D, T_w, \varepsilon_t, \varepsilon_w$. 