NEW

Ph.D. Qualifier Examination

Focus Area: Dynamics, Control and Mechatronics

Spring 2013

Rules:

- No Books, Notes and Calculators are allowed.
- Answer any 4 out of 6 questions.
- Time: 180 minutes.
- All problems are of equal weightage but unequal difficulty. So use your time wisely.

Pledge: On my honor, I have neither given nor received assistance on this examination.

Your signature:

P1: Consider the following system:

$$\dot{\mathbf{x}} = (A - BK)\mathbf{x} + Bu \tag{1}$$

where, A is a 4×4 constant matrix, B is a 4×1 constant column vector and K is a 1×4 constant row vector. Prove that the system $(A - BK, B)$ is controllable if and only if the system (A, B) is controllable.

P2: Consider the following two systems:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2}$$

$$y = [0\ 1]\mathbf{x} \tag{3}$$

and

$$\dot{\bar{\mathbf{x}}} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \bar{\mathbf{x}} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \tag{4}$$

$$y = [1\ -1]\bar{\mathbf{x}} \tag{5}$$

(a) Prove that both systems are similar by computing their transfer functions.

(b) Determine the similarity transformation that transforms the first system to the second system. Prove that your answer is correct by applying the similarity transformation to the first system to obtain the second system.

P3: A high performance helicopter has a model given by the equations:

$$\frac{d^2\theta}{dt^2} = -\sigma_1 \frac{d\theta}{dt} - \alpha_1 \frac{dx}{dt} + n\delta \tag{6}$$

$$\frac{d^2x}{dt^2} = g\theta - \alpha_2 \frac{d\theta}{dt} - \sigma_2 \frac{dx}{dt} + g\delta \tag{7}$$

where θ is the pitch angle of the helicopter, the control variable δ is the rotor angle, and x is the translation in the horizontal direction. The parameters of the model are:

$$\sigma_1 = 0.415, \sigma_2 = 0.0198, \alpha_1 = 0.0111, \alpha_2 = 1.43, n = 6.27, g = 9.8$$

(a) Determine the transfer function relating the pitch motion to the input δ.

(b) Assuming that one of the poles of the characteristic equation is at:

$$s = -0.6878$$

identify the remaining poles of the system.

(c) Assuming a proportional controller, sketch the root-locus. Can the proportional controller stabilize the system?

(d) Design a PD controller which generates a critically damped response.

P4: A fireman named p of weight W hangs on to the end of a ladder a that is telescoping out a ladder b at a constant linear speed $\mathbf{V}^{ab}$ as shown in the Figure 1. Simultaneously, b is rotating relative to the ladder c at a constant angular speed of $\dot{\gamma}$, while c is rotating relative to the turntable (disk) d at a constant angular speed $\dot{\beta}$ (a negative number - the β in the Figure 1 is drawn negative), and d rotates on the stationary fire engine f with a constant angular speed $\dot{\alpha}$. Ladders a,b and c are of length L, and at time $t = 0$, ladder a is telescoped wholly within b. Answer the following questions:

(a) Find the velocity $\mathbf{V}^{pe}$, the velocity of the fireman relative to the stationary Earth and express it in terms of the basis vectors $\hat{\mathbf{c}}_1, \hat{\mathbf{c}}_2$ and $\hat{\mathbf{c}}_3$ of Figure 1.

(b) Find the acceleration vector of the fireman in terms of the basis vectors $\hat{\mathbf{c}}_1$, $\hat{\mathbf{c}}_2$ and $\hat{\mathbf{c}}_3$.

(c) Draw a free body diagram of the fireman hanging from the ladder a, showing clearly all the forces acting on the fireman.

(d) Using Newton's second law of motion provide the equations of motion of the fireman.

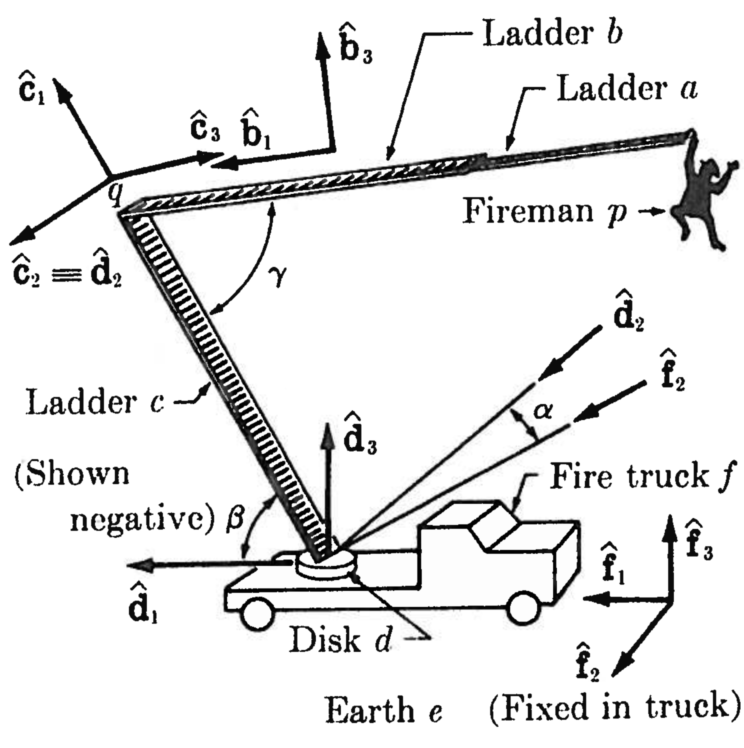


Figure 1: A schematic illustrating the fireman hanging on to a truck

P5: A nonlinear system is modeled by the following equation:

$$\ddot{y} + 0.01\dot{y} + \frac{dF(y)}{dy} = u \tag{8}$$

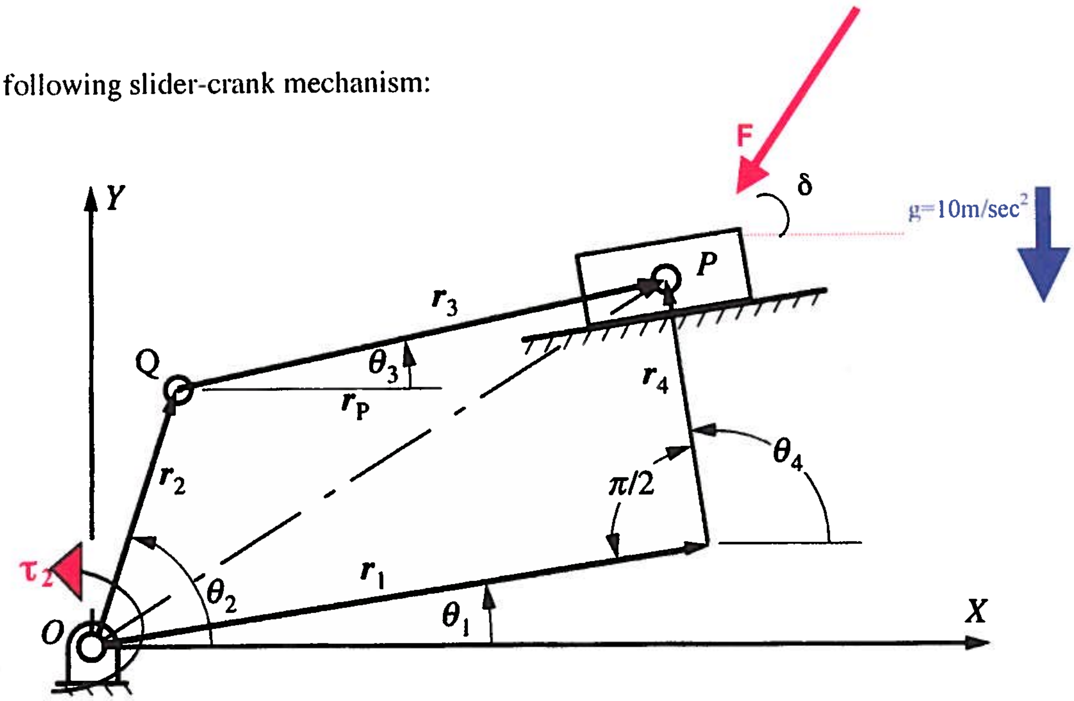
where $F(y) = (y^2 - 1)^2$, y is the system output and u is the system input. Your tasks are as follows:

(a) Find the equilibrium points of the system.

(b) If the system has any unstable equilibrium points, design a controller that will stabilize the system around each of these points.

P6:

For the following slider-crank mechanism:



	Ground Link	Input Crank	Coupler	Slider Offset
Lengths	r_1	$r_2 = 2.0$ m	$r_3 = 2.0$ m	$r_4 = 0.0$ m
Angles	$\theta_1 = 30°$	$\theta_2 = 75°$	θ_3	$\theta_4 = \theta_1 + 90°$

You are also given that:

- The moving links (crank, coupler) are assumed to be massless while the slider has a mass of 10kg.
- The linkage is set up in the vertical plane with gravity acting downwards as shown in the figure.
- You may assume that the gravitational constant is 10 m/sec^2.

For this system:

a) Compute the complete configuration of the system (r_1, θ_3) *using the loop-closure equations* for the *uncrossed* position.

b) Given that $\dot{\theta}_2 = 1$ rad/sec, compute the velocities of all the links in the system $(\dot{r}_1, \dot{\theta}_3)$.

c) Given that $\alpha_2 = \ddot{\theta}_2 = 0.1$ rad/sec^2, compute the acceleration of the slider.

d) Assuming that **Joint 2** is the actuated joint, calculate the torque (**t_2**) required to drive the system with the accelerations of part (c) when an external force **F** (of magnitude **50N**) acts on the slider at an angle $\delta =$ 60°, with respect to the horizontal.