January 2008

DESIGN AND OPTIMIZATION
Ph.D. Qualifying Exam

(One Hour, 45 Minutes)

There are a total of six problems divided into two categories, with three problems for each category: A) Optimization and B) Design. You **MUST** work two of the three problems in **EACH** category. In other words, work two problems from the set A1, A2, and A3 and then two more from the set B1, B2, and B3.

Please adhere to the following procedures:

Write your assigned number on your solutions but ***do not*** write your name.

Hand in ***both*** the solutions and examination questions.

If you are unable to complete a problem due to lack of a key equation or shortage of time, a clear explanation of how you would complete the problem should be made.

NOTE: You may have a book for each area available for reference during the exam.

A1. You are a design engineer in charge of optimizing production of your company's manufacturing facility. Your company makes two types of medical devices: A and B. Assuming that you can sell all of the products you make, you need to determine the rate which type of product should be produced.

Product A is a higher-end device, with more features. Therefore, only two of them can be produced each day. Product B is a lower-end device with fewer features. Three of them can be produced each day.

Producing either one requires the same amount of time in the electronics clean room. The clean room can handle four devices per day of either type. Based on market demand and current prices, current sales are generating \$15 per Product A produced and \$10 per Product B.

Please answer the following questions regarding this problem.

a) Construct a complete optimization formulation for the problem, identifying your objective function and all relevant constraints. Describe the problem formulation, explaining what kind of formulation is it, what kind of methods can be used to solve it.
b) Draw the design space, clearly labeling all constraints and objective function contours.
c) Solve the problem graphically, verifying that your solution is indeed optimal.
d) Set up the initial Simplex tableau for this problem, given a starting point of (0,0) and identify the first pivot point and resulting solution after one iteration.
e) In engineering design, many times the problem formulation is nonlinear. What relevance does linear programming have for nonlinear optimization problems? In other words, how can linear programming techniques be used to solve nonlinear optimization problems?

A2. Optimization Concepts (3 parts)

i. Figure a shows a 2-D constrained design space. Figures b and c go together and Figures d, e, and f go together. The space in Figures b and c represent the same optimization problem as in Figure a. The space in Figures d, e, and f represent the same optimization problem as in Figure a.

Describe what is being shown in Figures b and c. Describe what is being shown in Figures d, e, and f.

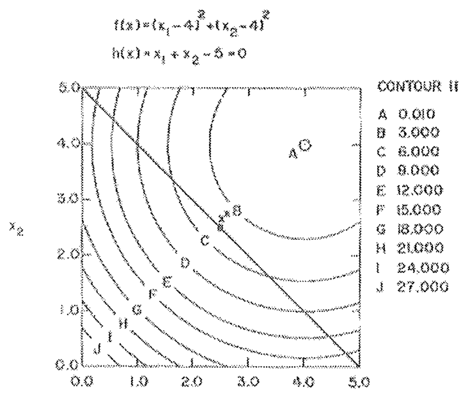


Figure a

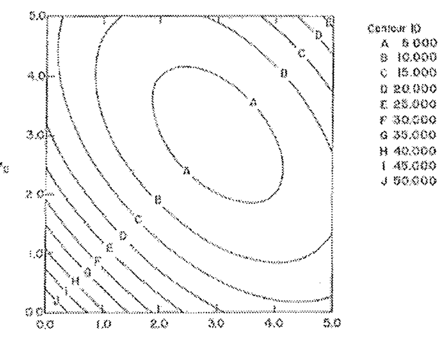


Figure b

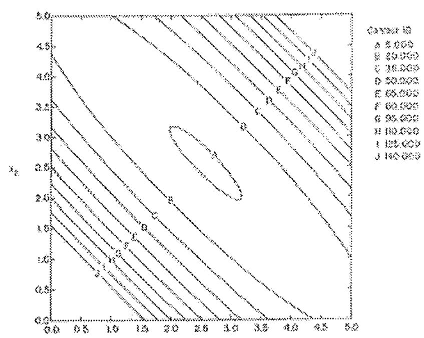


Figure c

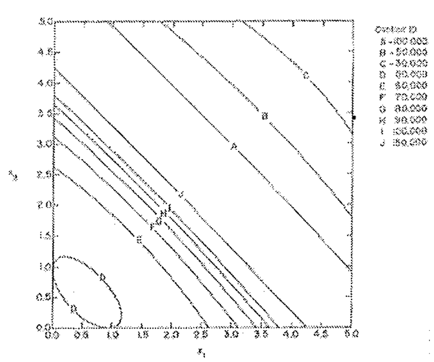


Figure d

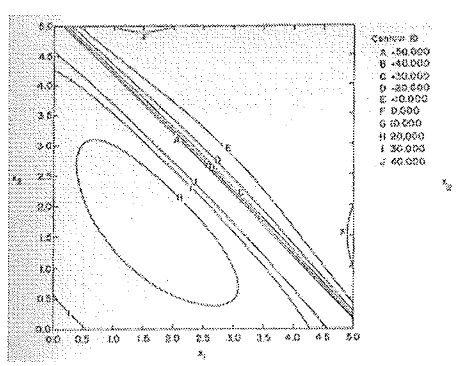


Figure e

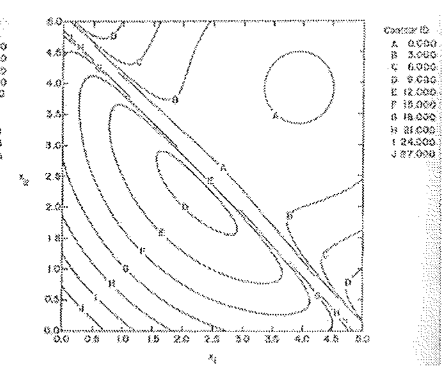


Figure f

ii. On the figure on the next page, start from a point (3,1) and clearly show where searches will result. Label appropriate final points as A, B, C, and D, associated with the following methods:

A. Method of Feasible Directions (1 search/iteration)
B. Method of Centers (1 search/iteration)
C. Sequential Linear Programming (1 search/iteration)
D. SLP with +/-1 move limits (2 searches/iterations)

Comment on the quality of the resulting solutions and suggest which would converge most quickly for the optimal design space shown.

A2. Optimization Concepts (3 parts – continued)

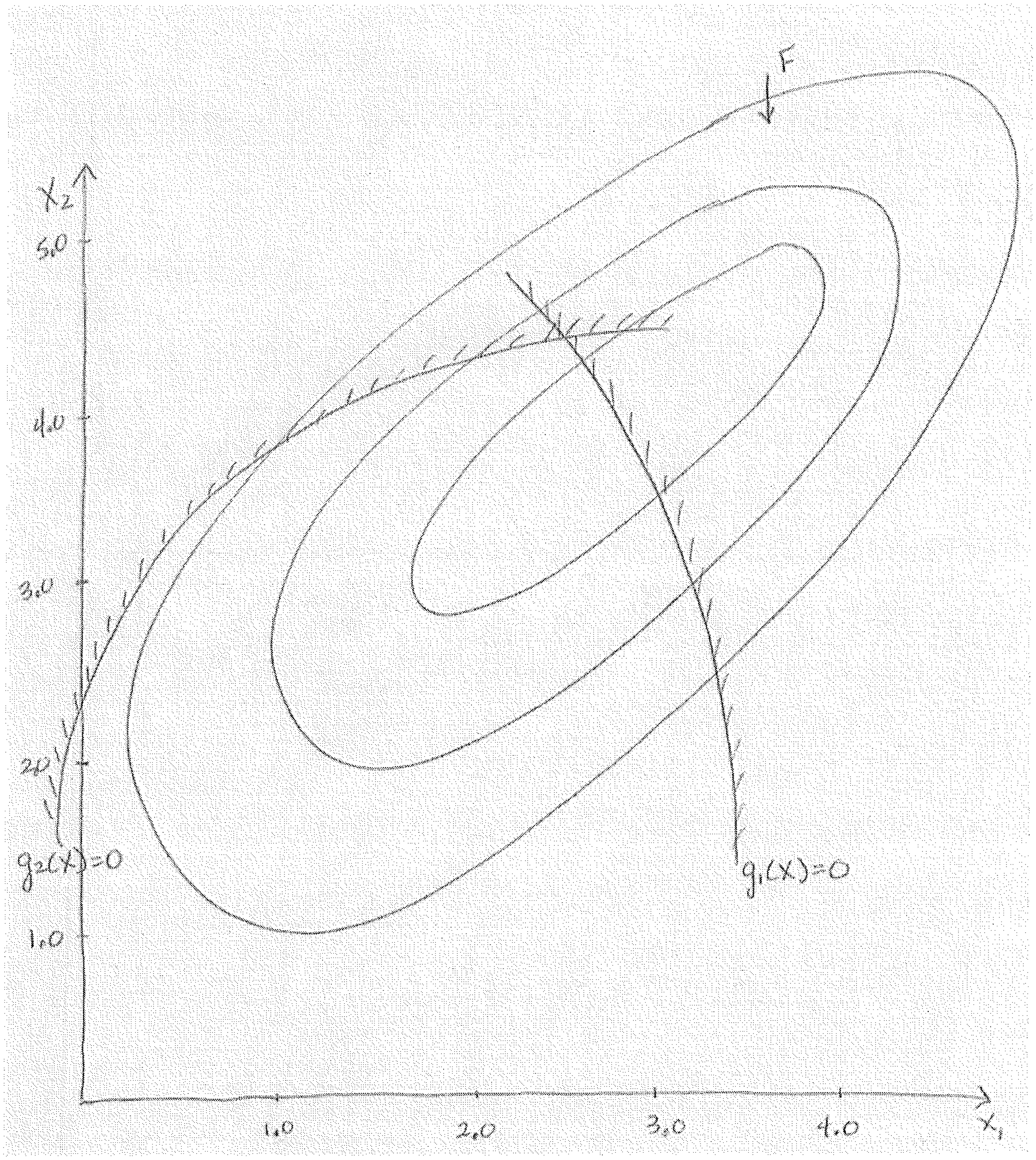


Figure for Problem A2, part ii.

A2. Optimization Concepts (3 parts – continued)

iii. Figure a shows a 2-D function in 3-D space (i.e. objective function in z direction) and Figure b shows the function projected onto the x1-x2 plane.

a) Use appropriate optimization terminology to describe the type of problem being illustrated;

b) Identify any potential issues that might exist when trying to find the 'best' solution; and

c) Suggest an appropriate approach for solving this problem from a random initial point.

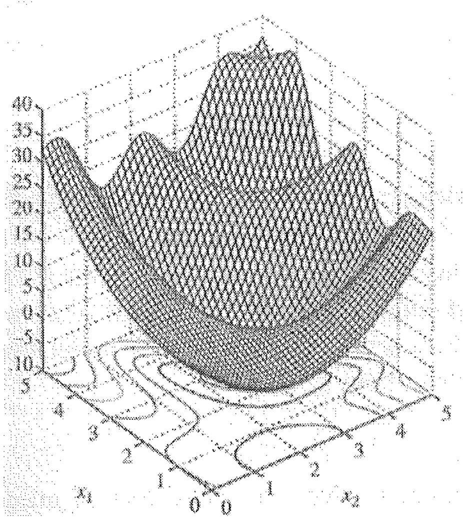


Figure a

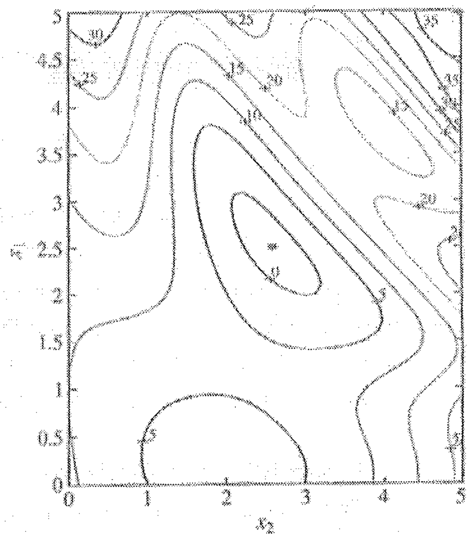


Figure b

A3. For the following optimization problem, please answer the associated questions.

Minimize $x_1 x_2$

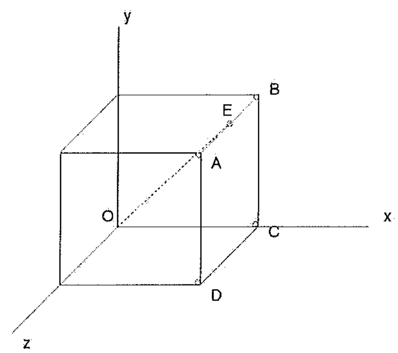
Subject to: $x_1 + x_2^2 \le 0$

$x_1^2 + x_2^2 \le 9$

a) Sketch the design space in the region, $-4 \le x_1, x_2 \le 4$, clearly labeling axes, constraints, and objective function contours.
b) Label where you think the optimal solution is for the problem on your graph. How would you classify this point from an optimality perspective?
c) Verify that the K-T conditions are met at the candidate solution (0,0). How would you classify this candidate solution for this problem? Is it the same solution or a different solution from your solution in part b)? Why?
d) Briefly discuss how you could apply two different methods to solve the problem – one being a direct method, and one a penalty method.

B1. Given the data points $P_0(0,0)$, $P_1(2,2)$, P_2 (3,1), $P_3(4, -1)$ and $P_4(5,3)$, describe a Bezier curve that will interpolate these points. The resulting Bezier control polygon must force the curve to pass through the given points.

B2. In the cube shown in the figure below, each side is of length 8 inches. Rotate this cube by an angle of 45 degrees counter clock wise about the diagonal OA, and find the new coordinates of E, A and D. Assume E is the mid point of AB.



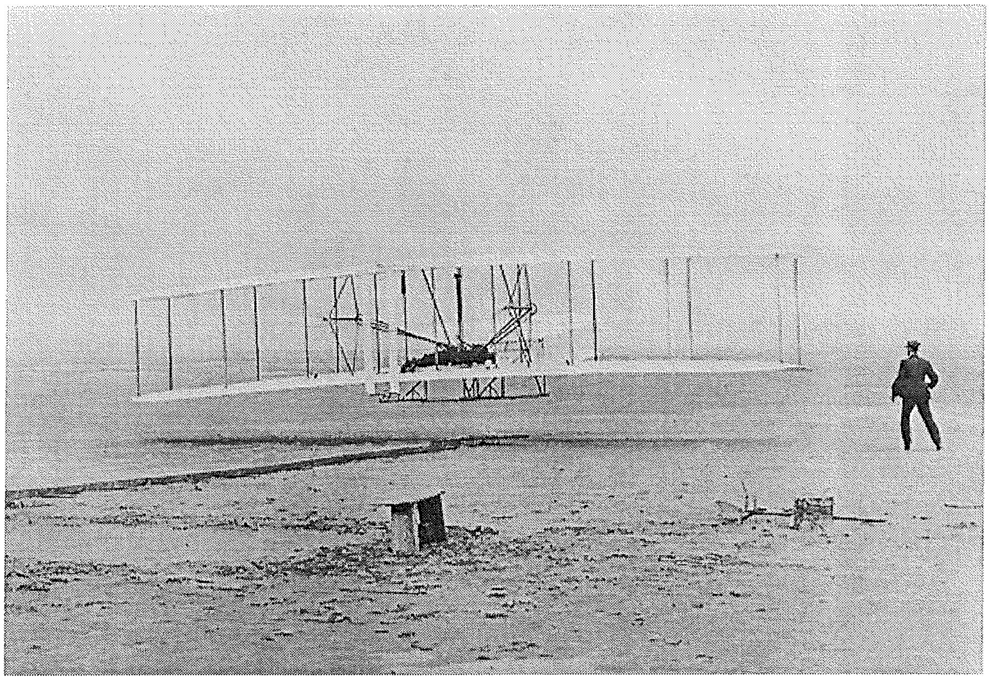
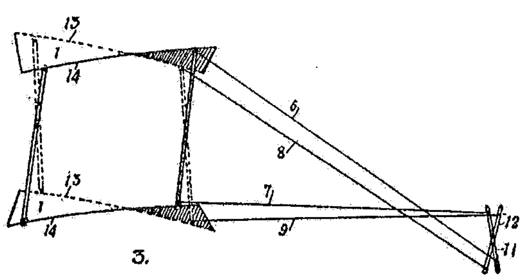
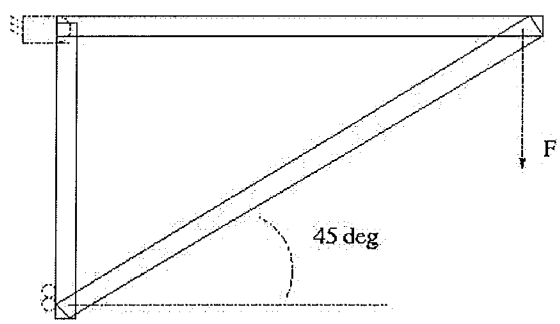


Figure for Problem B3

B3. In the pictures above and below you can see the original 1903 Wright brothers flyer and a sketch explaining the ingenious control mechanism where the pilot would "warp" the wings to bank (turn) the aircraft.

1a) Outline a plan for FEA of the structure – what elements would you use for the wings, the struts and the cables holding them together ? What loads would you apply (the Wrights got them from wind tunnel like testing but you will have to make them up)? Note the engine and the pilot.
1b) When the wing is warped and deflected, the strains can reach as high as 5%. What modeling assumptions should you use in the FEM code to account for this?
2) The stiffness matrix of a truss element is given as $[k]=(AE/L)[\{1\ -1\}^T\ \{1\ -1\}^T]$. For the truss structure shown below (right), assemble the appropriate stiffness matrix and force vectors and apply the boundary conditions.





Truss structure for part B3-2.