MAE CAM Ph.D. Qualifier (Tuesday, May 21, 2013)

Select five of the following eight problems. Maximum score for each selected problem is 20 pts.

Q1a. In continuum mechanics, it is assumed that the atomic structure of materials does not exist. Explain the underlying rationale by providing an example of a mathematical operation that will be meaningless if this assumption is not made. (4 pts)

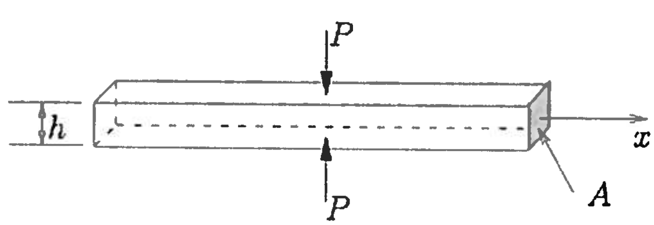
Q1b. Mark the following statements as "true" or "false". $(3\times 2\,\text{pts})$

a. A rigid body rotation (pure rotation) causes strain in a body.

b. A constitutive law, $\sigma = E\varepsilon^2$, is consistent with linear elasticity theory, where E is a material parameter.

c. Superposition principle is valid for strain energy, i.e., strain energy produced by multiple loads acting simultaneously is simply equal to the sum of the strain energies associated with the loads acting separately.

Q1c. Consider a slender uniform bar compressed by two equal and opposite forces P as shown in the following figure. Prove that the total elongation δ of the bar in x direction is given by $\delta = \nu Ph/AE$, where A, ν, and E are, respectively, cross-sectional area, Poisson's ratio, and Young's modulus of the bar. (10 pts)



Q2. The governing differential equations for a Timoshenko beam can be written in the following form:

$$EI\frac{d^2\beta}{dx^2} + GAk\left(\frac{dw}{dx} - \beta\right) = 0, \quad GAk\left(\frac{d^2w}{dx^2} - \frac{d\beta}{dx}\right) + p = 0$$

where x is the axial coordinate, w is the lateral deflection and

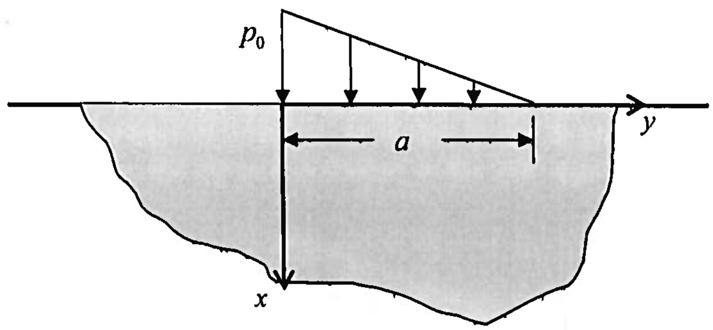
$$\beta = \frac{dw}{dx} - \gamma$$

with shear strain γ assumed constant throughout each cross-section. Additionally, E, G, I and A represent the elastic modulus, shear modulus, cross-sectional moment of inertia and cross-sectional area, respectively, while k is a parameter to compensate for the actual non-uniformity of shear stress through the section.

Derive the principle of virtual work for this Timoshenko beam of length L and formulate a two-noded finite element based upon that principle. Finally, briefly discuss the difficulties that one faces when using this finite element to model slender beams (i.e., beam with very large length-to-depth ratios).

Q3. Consider a point A within a three-dimensional linear elastic isotropic body, undergoing isothermal deformations. The stress and strain at that point are given by the tensor components σ_{ij} and ε_{ij}, respectively. Determine the relationships between the principal stresses and principal strains and their corresponding principal directions.

Q4. Consider an isotropic elastic half-space subjected to a linearly varying pressure distribution applied on a portion of the surface of width a, as shown in the diagram. Formulate the problem with the objective to determine the stress distribution in the half-space.



The following formulas may be helpful.

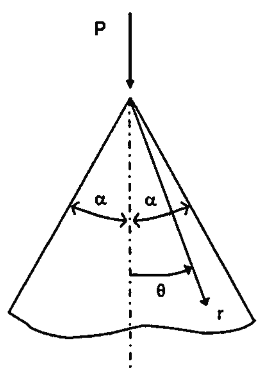
Wedge Loaded at Vertex:

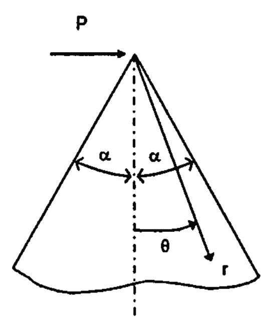
$$\sigma_{rr} = -\frac{2P}{2\alpha + \sin(2\alpha)} \frac{\cos\theta}{r}$$

$$\sigma_{\theta\theta} = \sigma_{r\theta} = 0$$

$$\sigma_{rr} = -\frac{2P}{2\alpha - \sin(2\alpha)} \frac{\sin\theta}{r}$$

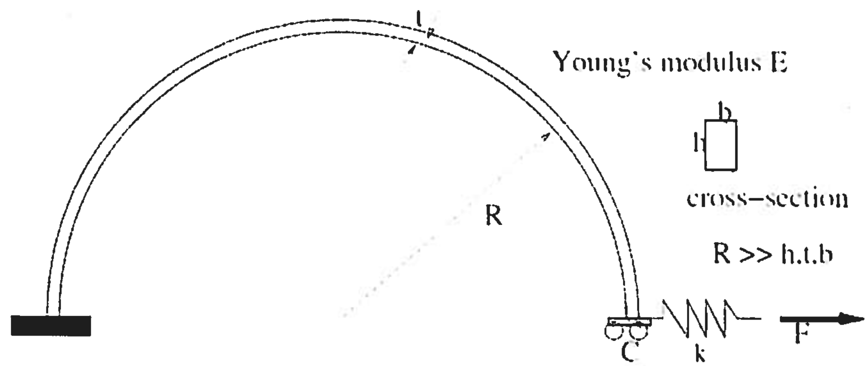
$$\sigma_{\theta\theta} = \sigma_{r\theta} = 0$$





Q5.

Use an energy method to compute a relationship between the force F and the deflection of the point C on the thin semicircular arch shown below.



Q6.

After the recent snow storm in Buffalo, John Q. Engineer was asked by his "boss" to compute the possibility of his house roof collapsing. John has determined that the roof rests on a pair of simply supported beams (like those shown above) made of "swill" which has the stress-strain law shown above. He assumes that for the roof to collapse some point in the beam must attain a stress equal to the yield stress σ_{yp}. Please help John by answering the following questions:

a) Compute the collapse load (value of q) at which some point in the beam is at a stress $\sigma = \sigma_{yp}$.If the density of snow is ρ, acceleration due to gravity is g and the area of the roof is A what is the the depth of snow on the roof h that may be supported before the roof collapses ?

b) What is the maximum depth of the snow that can be supported before the beam is permanently deformed ?

c) If the beam is at it's maximum elastic deformation, what is the curvature of the beam ?

Q7. The governing differential equation for an isotropic elastic plate under the Kirchhoff-Love theory can be written in the following form:

$$D\,\nabla^4 w = p,$$

where w is the transverse deflection, p is the pressure, ∇^4 is the biharmonic operator and

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

with E, ν and h representing the elastic modulus, Poisson ratio and the plate thickness, respectively.

Derive the principle of virtual work for a simply-supported square plate of dimension $L \times L \times h$.

Q8.

Examine the interference fit between two elastic circular tubes, both having elastic modulus E and Poisson ratio ν. In the undeformed condition, tube A has inner radius R and outer radius $2R$, while tube B has inner and outer radii $2R - \delta$ and $4R$, respectively. Thus δ represents the interference. Assuming that $\delta \ll R$, determine the maximum hoop stress $\sigma_{\theta\theta}$ in the tubes. Neglect friction.

The following formulas for a pressurized thick circular cylinder under conditions of generalized plane stress may be useful. Here R_1 and R_2 represent the inner and outer radii, respectively. Meanwhile, p_1 and p_2 denote the internal and external pressures, respectively.

$$\sigma_{rr}(r) = \frac{c_1}{r^2} + c_2 \qquad c_1 = \frac{R_1^2 R_2^2 (p_2 - p_1)}{R_2^2 - R_1^2}$$

$$\sigma_{\theta\theta}(r) = -\frac{c_1}{r^2} + c_2 \qquad c_2 = \frac{R_1^2 p_1 - R_2^2 p_2}{R_2^2 - R_1^2}$$

$$\sigma_{r\theta} = 0$$

$$\sigma_{zz} = 0, \quad \sigma_{zr} = 0, \quad \sigma_{z\theta} = 0$$

$$\epsilon_{rr} = \frac{1}{E}[\sigma_{rr} - \nu\sigma_{\theta\theta}] = \frac{du_r}{dr}$$

$$\epsilon_{\theta\theta} = \frac{1}{E}[\sigma_{\theta\theta} - \nu\sigma_{rr}] = \frac{u_r}{r}$$

$$\epsilon_{zz} = -\frac{\nu}{E}[\sigma_{rr} + \sigma_{\theta\theta}]$$

$$\epsilon_{r\theta} = 0, \quad \epsilon_{rz} = 0, \quad \epsilon_{\theta z} = 0$$