Question CAM_1

The governing differential equations for a Timoshenko beam can be written in the following form:

$$EI\frac{d^2\beta}{dx^2} + GAk\left(\frac{dw}{dx} - \beta\right) = 0\,, \quad GAk\left(\frac{d^2w}{dx^2} - \frac{d\beta}{dx}\right) + p = 0$$

where x is the axial coordinate, w is the lateral deflection and

$$\beta = \frac{dw}{dx} - \gamma$$

with shear strain γ assumed constant throughout each cross-section. Additionally, E, G, I and A represent the elastic modulus, shear modulus, cross-sectional moment of inertia and cross-sectional area, respectively, while k is a parameter to compensate for the actual non-uniformity of shear stress through the section.

Derive the principle of virtual work for this Timoshenko beam of length L and develop a two-noded finite element based upon that principle. Finally, briefly discuss any difficulties that one might face when using this finite element to model slender beams (i.e., beam with very large length-to-depth ratios).

Question CAM_2

Consider a point A within a three-dimensional linear elastic isotropic body, undergoing isothermal deformations. The stress and strain at that point are given by the tensor components σ_{ij} and ε_{ij}, respectively. Determine the relationships between the principal stresses and principal strains and their corresponding principal directions.

Question CAM_3

Consider the problem of heat conduction along a solid rod of length L governed by the partial differential equation

$$\frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} + h(U - u)$$

where $u(x,t)$ represents the temperature, while κ is the thermal diffusivity.

The second term on the right-hand-side models convective heat transfer between the rod and a surrounding fluid medium having constant uniform temperature U. The parameter h represents the heat transfer coefficient, which is assumed to be a positive constant.

Solve the initial/boundary value problem under the following conditions:

$$u(x,0) = 0, \quad u(0,t) = 0, \quad u(L,t) = U$$

Expansion coefficients may be expressed in integral form. However, sketch approximate temperature distributions along the length of the rod at various times and specify the expected steady-state solution.

Question CAM_4

Assume that the simple dynamic system shown below starts from rest and is subjected to a known acceleration at the left-hand support a_s that may vary over time. Develop an explicit time domain formulation to obtain an approximate solution in terms of the time step Δt. Additionally, discuss any limitations on Δt necessary to maintain stability of the numerical solution.