Department of Mechanical and Aerospace Engineering
2008 Ph.D Qualifying Exam
Jan 28, 2008

Dynamics and Control

Assigned #__________

NO TEXTBOOKS OR NOTES ALLOWED. YOU ARE REQUIRED TO ANSWER 4 OF THE 6 QUESTIONS.

Problem A:

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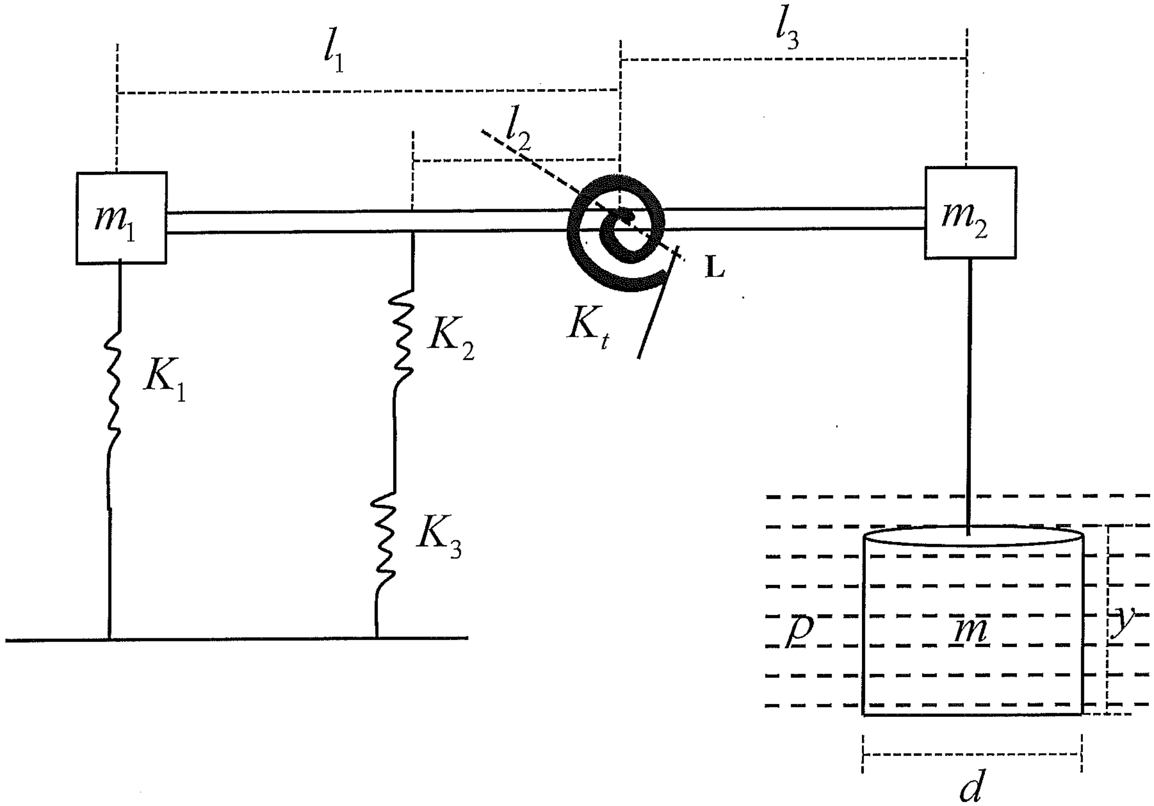
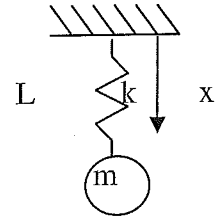


What is the damping ratio and the natural frequency of the system?

Problem B:

Consider the spring mass system shown below. The spring has a mass density of δ kg/m in its un-deformed state, and an un-deformed length of L. Derive the equations of motion and determine the natural frequency of the system.



Problem C:

Consider the following partitioned system matrix:

$$A = \begin{bmatrix} A_{11}(t) & A_{12}(t) \\ 0 & A_{22}(t) \end{bmatrix}$$

where $A_{11}(t)$, $A_{22}(t)$ and $A_{12}(t)$ are generally time-varying sub-matrices (they cannot be assumed to be scalars).

a) Prove that the state transition matrix has the form

$$\Phi(t,t_0) = \begin{bmatrix} \Phi_{11}(t,t_0) & \Phi_{12}(t,t_0) \\ 0 & \Phi_{22}(t,t_0) \end{bmatrix}$$

where $(\partial/\partial t)\Phi_{ii}(t,t_0) = A_{ii}(t)\Phi_{ii}(t,t_0), i = 1,2$ and
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b) Use this result to find $\Phi(t,0)$ for

$$A = \begin{bmatrix} -1 & e^{2t} \\ 0 & -1 \end{bmatrix}$$

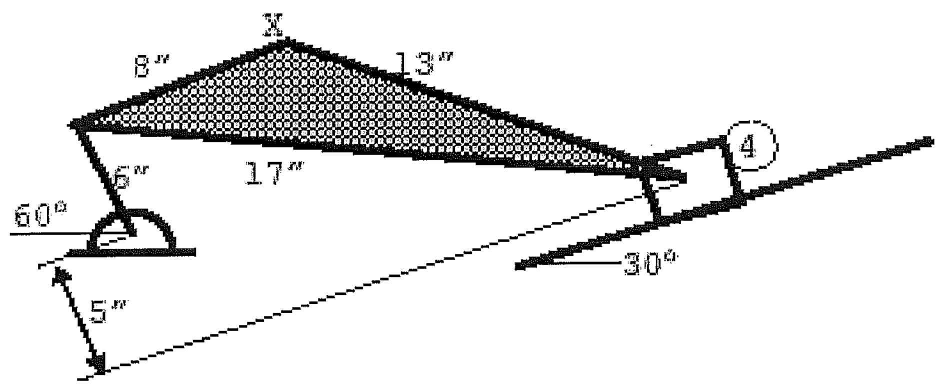
Prove that your answer is correct.

Problem D:

Consider the four-link mechanism shown in the figure below.
Determine the maximum displacement of link 4.

The mechanism is driven by a D.C. motor attached to the crank with an angular velocity of 60 rpm clockwise.

Determine the velocity of the link 4 when the crank rotates at 60 rpm for the link locations shown below.



Problem E:

Consider the following open-loop system:

$$\frac{C(s)}{U(s)} = \frac{1}{s^2 - 1}$$

- With an impulse input for $u(t)$ and zero initial conditions, solve for $c(t)$.
- Draw the Bode plot for the open-loop system.
- Suppose we multiply the open-loop transfer function by a constant. What does this do to the magnitude and phase of the Bode plot?
- Can the system be controlled from an asymptotic sense using proportional control only? Explain this answer using 1) a phase margin argument, as well as 2) from the closed-loop tansfer.
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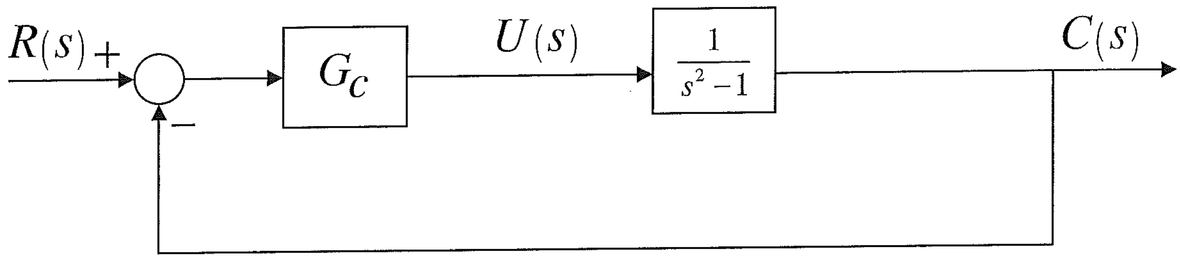

Problem F:

A linear, time-invariant, SISO (single input – single output) system is represented by the Bode plots shown below. Answer the following questions:

a) Estimate the system's transfer function
b) Is this system stable, marginally stable, or unstable? Explain.
c) What is the system's output if the input is u = 5 * sin (4t)?
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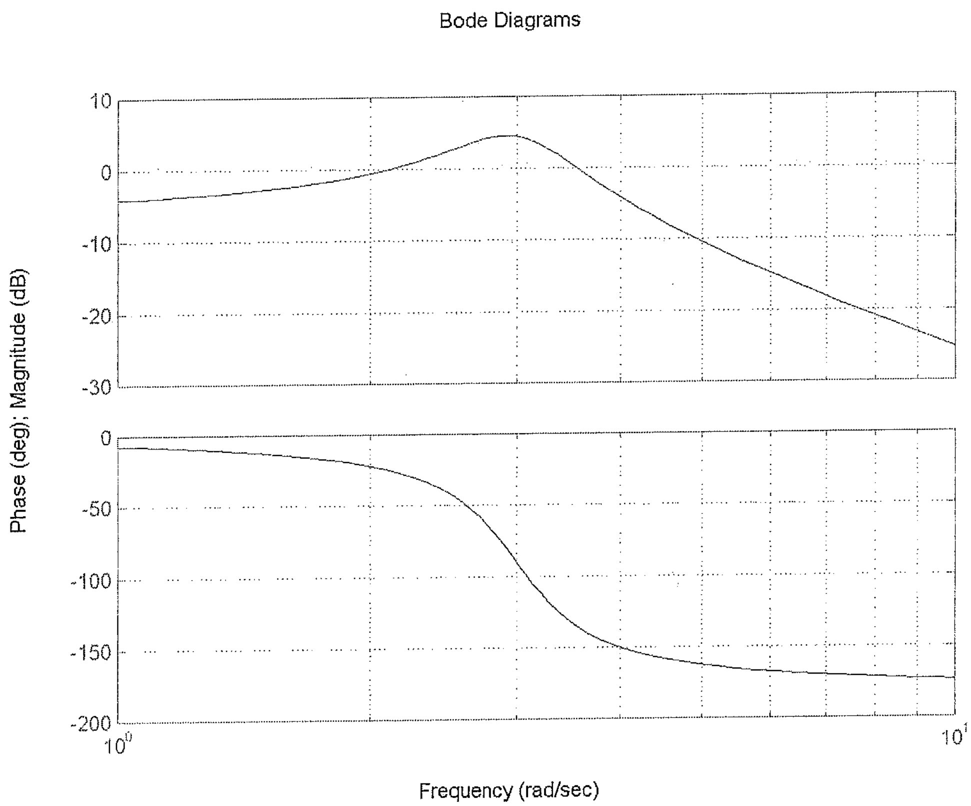
What is the system's approximate damping ratio?



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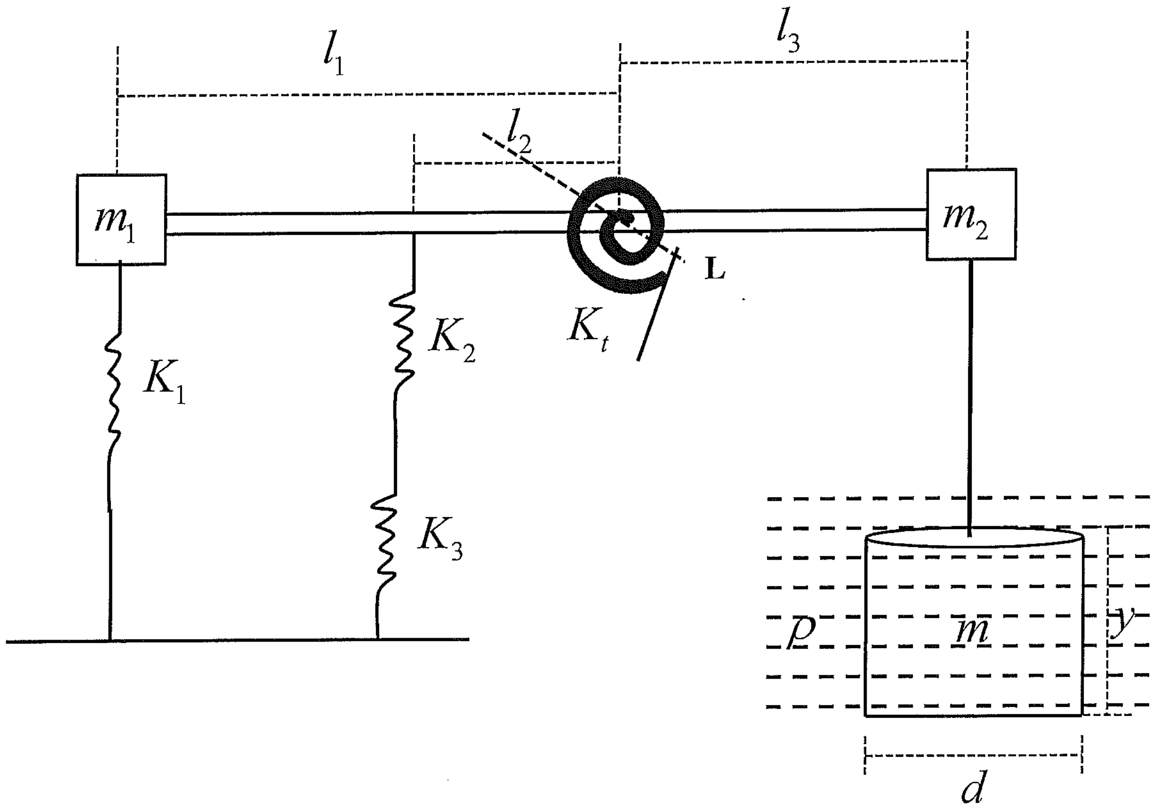
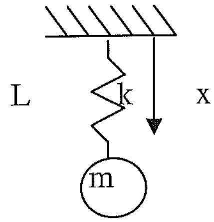


What is the damping ratio and the natural frequency of the system?

Problem B:

Consider the spring mass system shown below. The spring has a mass density of δ kg/m in its un-deformed state, and an un-deformed length of L. Derive the equations of motion and determine the natural frequency of the system.



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Consider the four-link mechanism shown in the figure below.
Determine the maximum displacement of link 4.

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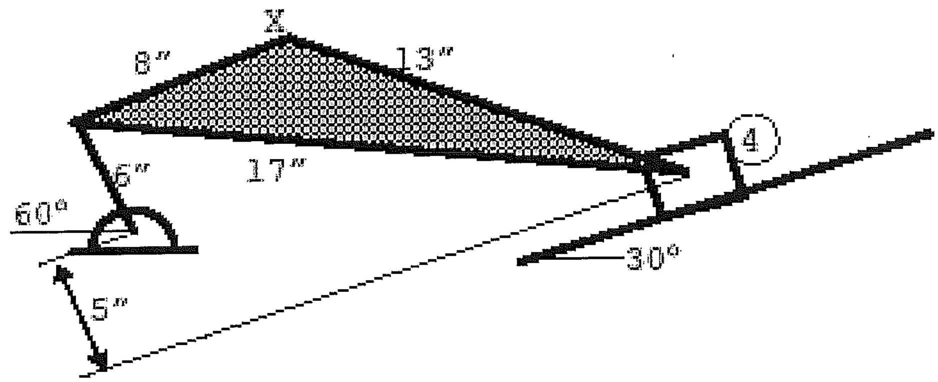

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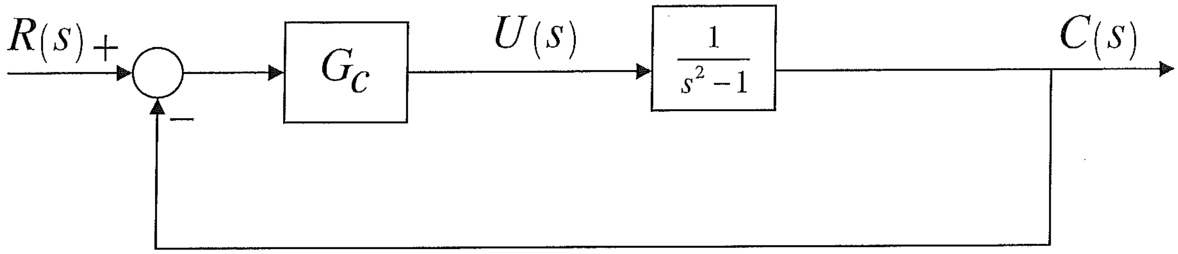

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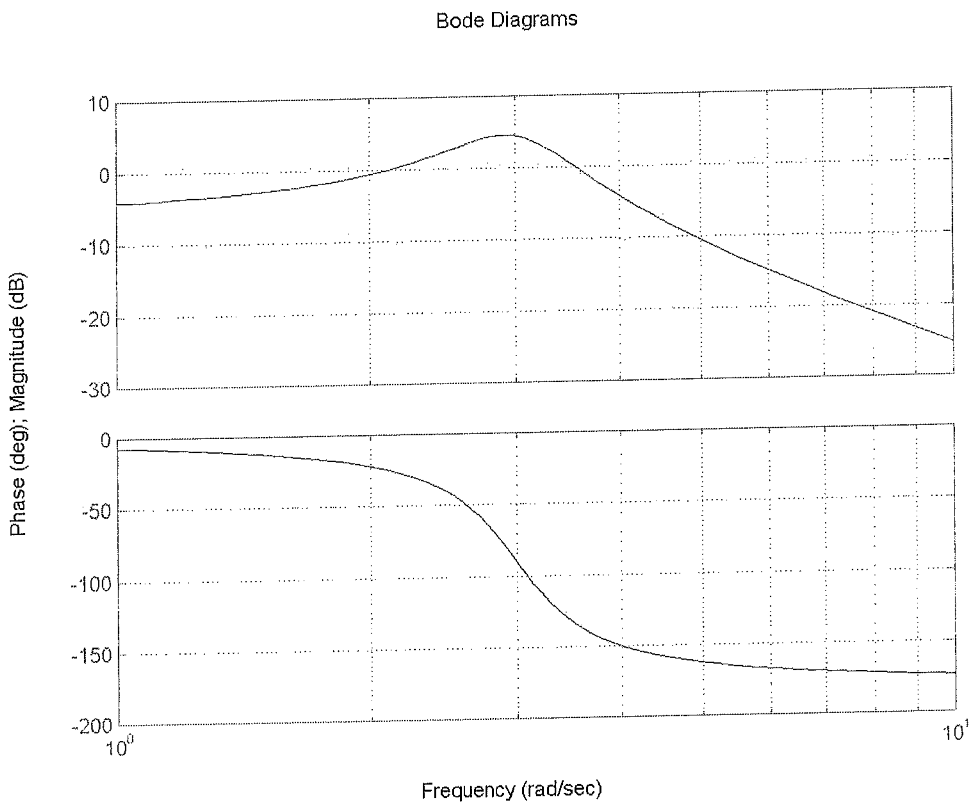

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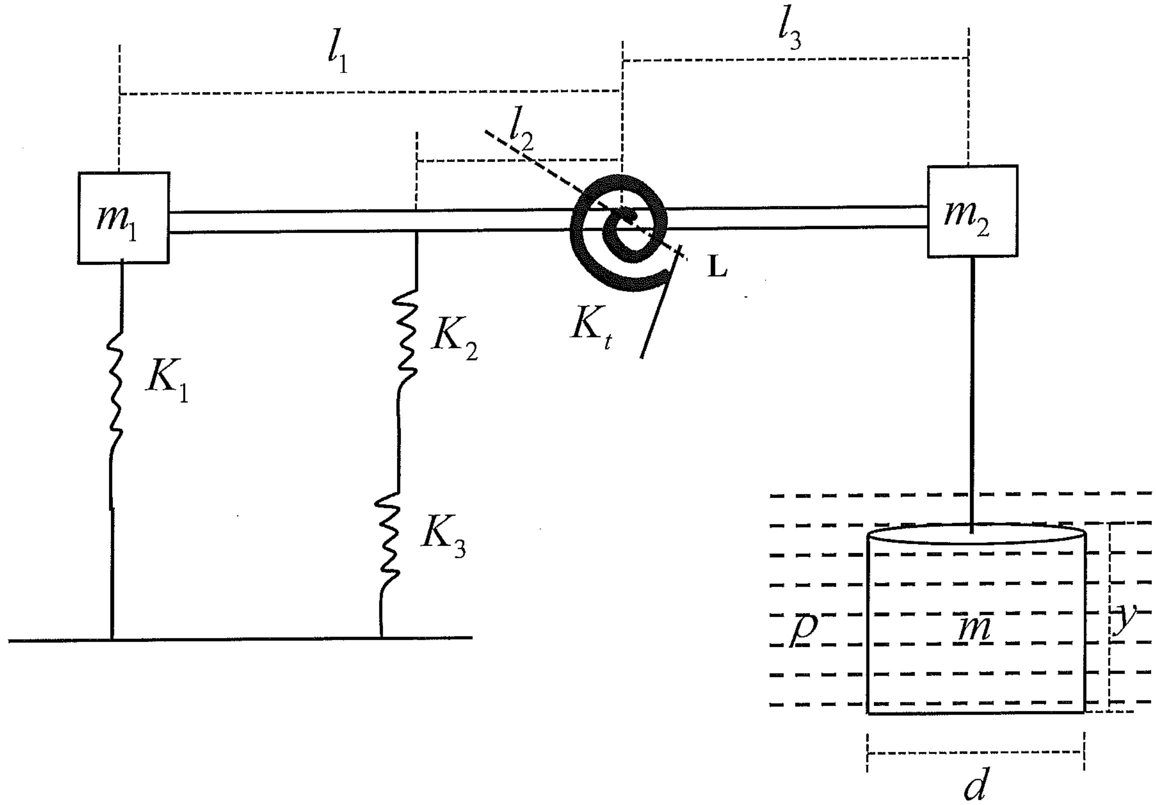

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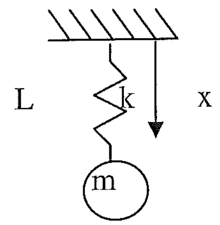

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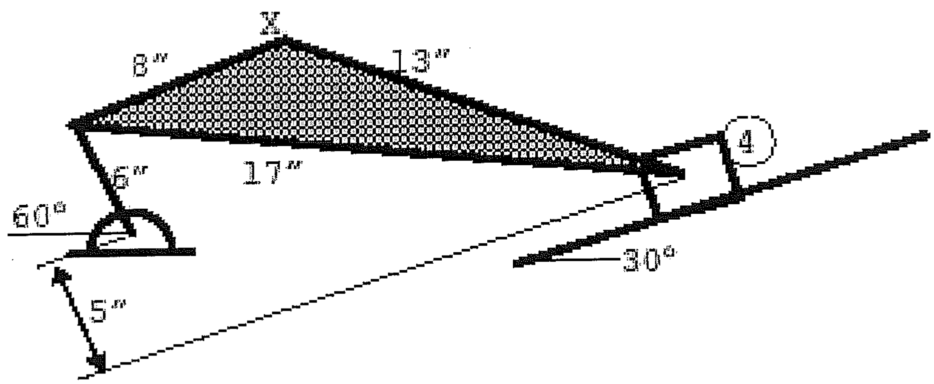

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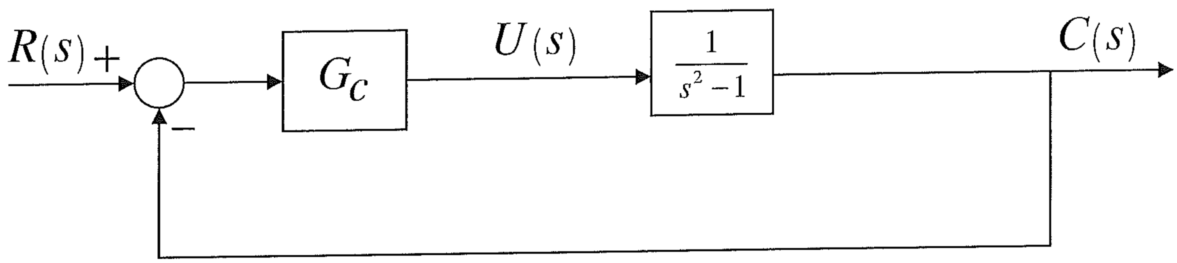

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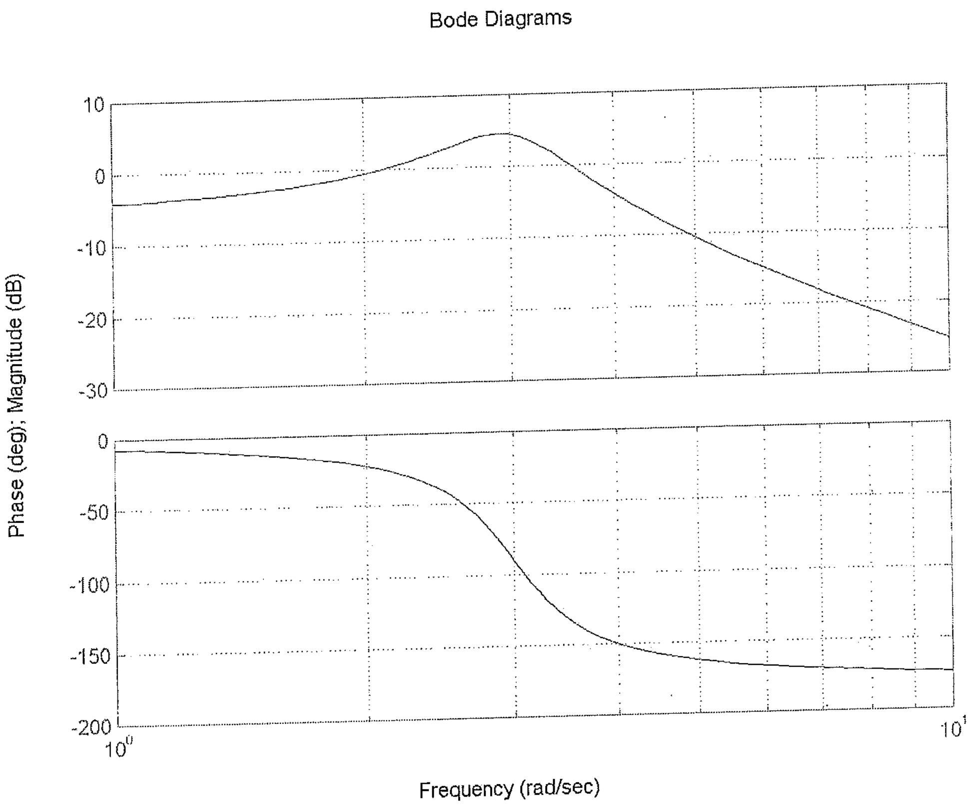

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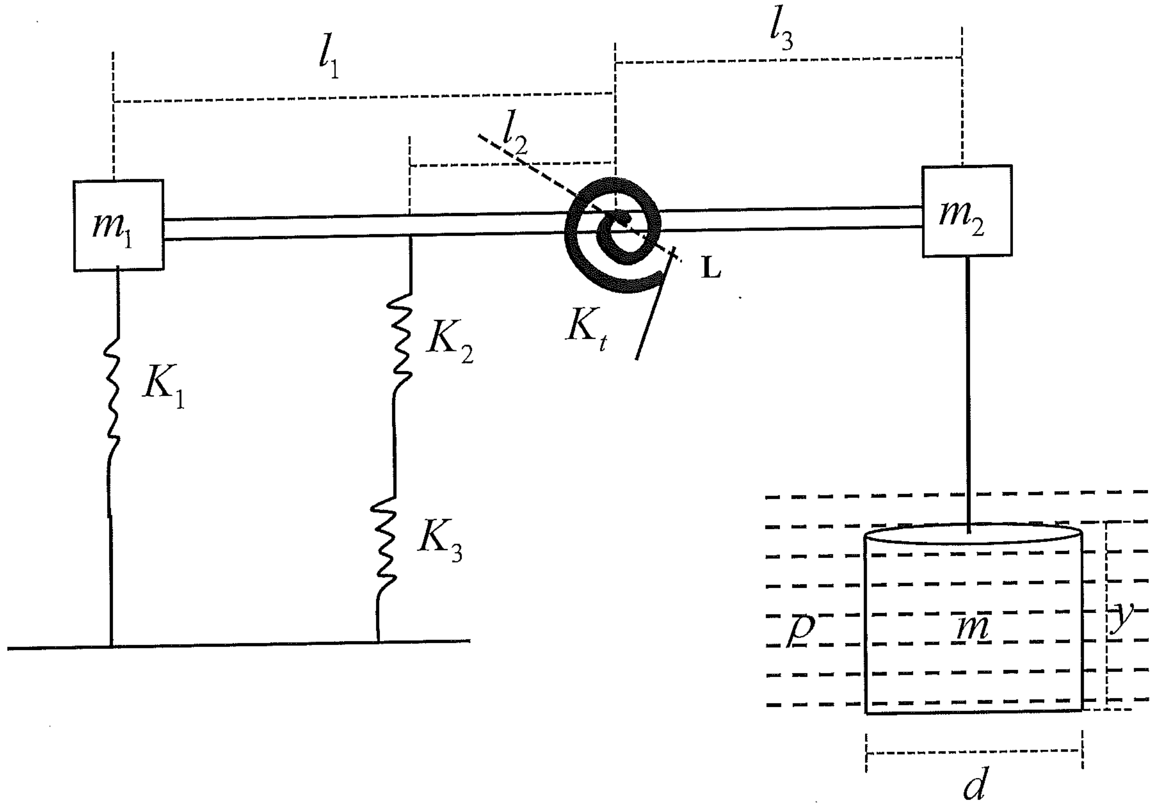

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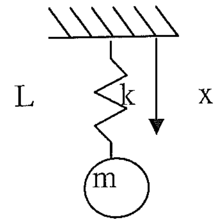

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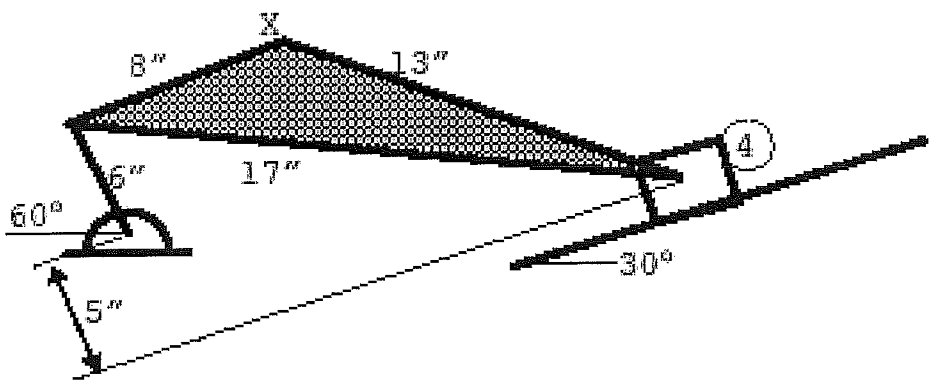

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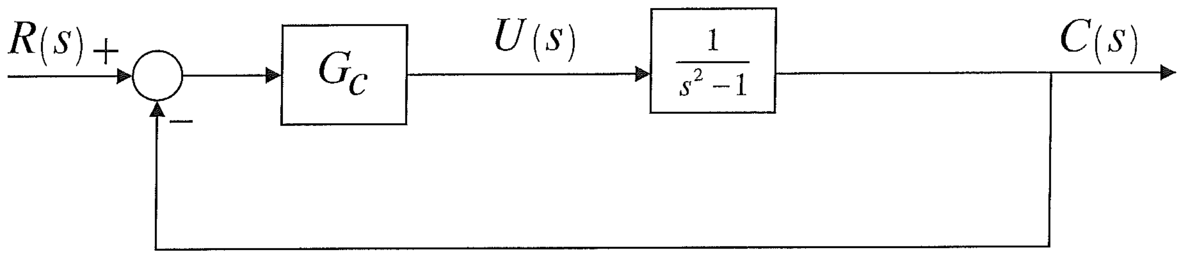

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