May 2008

SYSTEMS AND DESIGN
Ph.D. Qualifying Exam

(Three Hours)

There are a total of six problems divided into two categories, with three problems for each category: A) Systems and B) Optimization. You must work **any four of the six** problems provided.

Please adhere to the following procedures:

Write your assigned number on your solutions but ***do not*** write your name.

Hand in ***both*** the solutions and examination questions.

If you are unable to complete a problem due to lack of a key equation or shortage of time, a clear explanation of how you would complete the problem should be made.

NOTE: You may have a book for each area available for reference during the exam

SYSTEMS

A1. Find the state transition matrix of the following system:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & t \\ 0 & 2 \end{bmatrix} \underline{x}$$

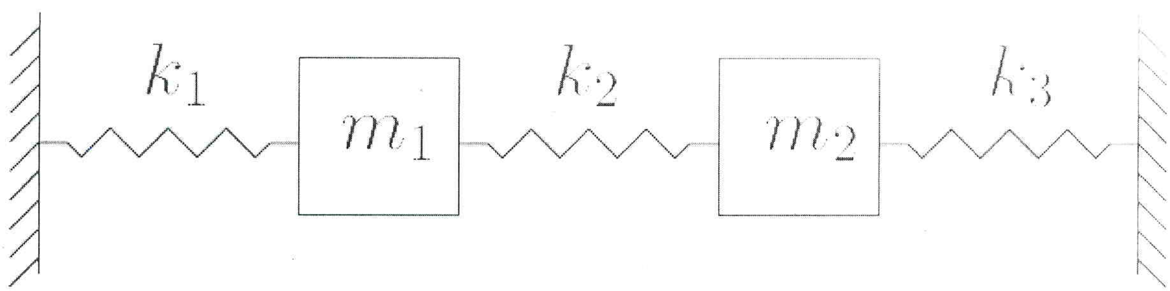
A2. Consider the following system matrices:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} a & 1 & b \end{bmatrix}$$

a) Find the eigenvalues of this matrix.

b) Mathematically show the observability of the system in terms of a, b or both.

A3. Consider the following spring-mass system with two masses m_1 and m_2 are suspended between three identical springs with spring constants k_1, k_2 and k_3 as shown in the figure below.



Let $\mathbf{x}(t)$ is a vector of horizontal displacements of mass m_i at time t. Find matrices $\mathbf{M}$ and $\mathbf{K}$ so that equations of motion are

$$\mathbf{M}\ddot{\mathbf{x}}(t) = -\mathbf{K}\mathbf{x}(t)$$

Further, we would like to find a solution of the form $\mathbf{x}(t) = e^{i\omega t}\mathbf{v}$ for constant vector $\mathbf{v}$. Show that this reduces to the problem of solving an algebraic equation of the form

$$\mathbf{K}\mathbf{v} = \lambda\mathbf{M}\mathbf{v}$$

This is called a generalized eigenvalue problem. Find the generalized eigenvalues for $m_1 = 1, m_2 = 2, k_i = 1$. Describe the two corresponding modes of vibration.

OPTIMIZATION

B1. Consider the optimization problem described below.

Enterprising 23 year old mechanical engineering students have set up a small business. They can produce 225 bottles of pure alcohol each week. They bottle two products from alcohol: 1) wine (20 proof), and 2) whiskey (80 proof). Pure alcohol is 200 proof. They have an unlimited supply of water but can only obtain 800 empty bottles per week because of still competition. The weekly supply of sugar is enough for 600 bottles of wine or 1200 bottles of whiskey. They make \$2.00 profit on each bottle of wine and \$4.00 profit on each bottle of whiskey. They sell whatever they produce (to appropriately aged responsible buyers).

a) Formulate the optimization problem in standard form.

b) Graphically solve the problem to determine how many bottles of wine and whiskey they should produce each week to maximize profit (be sure to label appropriately).

c) Verify K-T conditions at the solution points.

d) Sketch a close up of what is happening at the global optimum and clearly show gradients of objective function and constraint functions on the graph. Explain the graphical implications of the K-T conditions using your sketch.

B2. You are the Product Design Manager for Global Design, Inc. Two of your best engineers have been trying to model the behavior of a cam-follower mechanism.

One engineer claims that the distance the cam follower travels, y, is a function of time, t, as:

$$\mathbf{y = (t^2 - 1)/t}$$

while the other engineer claims that the distance is a function of time as:

$$\mathbf{y = t + \ln(t - 2)}$$

Not wanting to hurt either's feelings, you tell them that a combination of the behaviors can be used to model the cam follow. The first step, however, is to find out how similar the two functions are. To do this, you need to determine at what values of t the two functions are the same. In order to do this, you must solve the set of equations given above. Thinking back to your optimization course, you realize you can use simple optimization techniques to solve this problem.

a) State this equation solving problem in the form of an optimization problem in standard form.

b) Solve your optimization problem by applying any technique you want (learned in MAE 550). Please perform only 2 iterations of the method you choose.

c) What other approaches could you use to solve this problem? List three and provide an advantage and disadvantage of each.

B3. You are given a 2-D nonlinear optimization problem as shown in the figures below.
NOTE: This tests conceptual understanding – don't waste time trying to calculate anything.

a) Identify all optimal points (local or global) with X*.
b) From starting point A, please sketch the first 2 moves in a Steepest Descent approach.
c) From starting point B, please sketch the first cycle in a Powell's Conjugate Search approach (involves 3 directions).
d) If you used C as a starting point with a Powell's approach, mark the point at which you think the method would converge to a solution with XC.
e) How could you be assured of finding the global optimum in this problem? Where is the global optimum?